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New Results of Fuzzy Alpha - Open Sets Fuzzy Alpha - Continuous Mappings

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Abstract

This paper is devoted to introduce new definitions of fuzzy α- open set and new weaker forms of fuzzy α- continuous mappings. Properties and relationship between fuzzy α- open sets and fuzzy α- continuous mappings and other weaker form of them are investigated.

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Keywords: fuzzy α-open set; fuzzy α-continuous mapping; fuzzy $\alpha^{\star}$-continuous mapping; fuzzy $\alpha^{\star\star}$-continuous mapping

1 Introduction

In [[8], [2]], Singal, Rajvanshi and Bin Shahna have introduced the concept of fuzzy $\alpha - open$ sets. Several notions based on fuzzy α-open(closed) sets and fuzzy α- continuous mapping have been studied. Moreover, the study also included the relationship between those concepts and some other weaker forms of fuzzy open sets and fuzzy continuous mappings. In this paper, we introduce new definitions of fuzzy α- open set and new weaker forms of fuzzy α- continuous mappings and study several properties of $\alpha-$ open sets and α-continuous mappings in fuzzy setting and discuss their relations with other weaker forms of fuzzy continuous mappings.

2 Preliminaries

Throughout this paper by (X, τ) or simply by X we mean a fuzzy topological space (fts, shorty) and $f : X \rightarrow Y$ means a mapping f from a fuzzy topological space X to a fuzzy topological space Y. If u is a fuzzy set and p is a fuzzy singleton in X then $N(p)$, $Intu$, clu, u^c denote respectively, the neighborhood system of p, the interior of u, the closure of u and complement of u.

Now we recall some of the basic definitions in fuzzy topology.

Definition 2.1 [4] *A fuzzy singleton p in X is a fuzzy set defined by: $p(x) = t$, for $x = x_0$ and $p(x) = 0$ otherwise, where $0 < t \leq 1$. The point p is said to have support x_0 and value t.*

Definition 2.2 *A fuzzy set u in a fts X is called: Fuzzy feebly open [6] (resp. Fuzzy $\alpha-$open, Fuzzy preopen, Fuzzy $\beta-$open, Fuzzy semi-preopen, Fuzzy regular open, Fuzzy semi α-open)set if there is a fuzzy open set m such that $m \leq u \leq scl\ m$ (resp. $u \leq IntclIntu$, $u \leq Intclu$, $u \leq clIntclu$, there exists a fuzzy preopen set m such that $m \leq u \leq clm$, $u = Intclu$, there exists a fuzzy α-open set m in X such that $m \leq u \leq clm$) where $scl\ u$ is fuzzy semi closure u, is defined by the intersection of all fuzzy semi closed sets containing u. The family of all fuzzy α-open sets of X is denoted by $F\alpha O(X)$ and the family of all fuzzy semi α-open sets of X is denoted by $FS\alpha O(X)$.*

3 Fuzzy α-open set

In this section, we will present an equivalent definition to fuzzy α-open set and prove many spacial properties of it. Moreover, we will explain the relationship between different classes of fuzzy open sets by diagram.

Definition 3.1 *a fuzzy $\alpha-$interior of a fuzzy set u in X is denoted by:*
$Int_\alpha u = \vee\{v : u \leq v, v$ *is a fuzzy α-open set* $\}$

From Definition (3.1), we can prove this proposition.

Proposition 3.2 *Let X be a fts, $u \leq v \leq X$ then :*
(i) $Int_\alpha u$ *is a fuzzy $\alpha-$open set in* (X, τ).
(ii) $Int_\alpha u \leq u$.
(iii) $Int_\alpha u \leq Int_\alpha v$.
(iv) $Int_\alpha u = Int_\alpha(Int_\alpha u)$.
(v) *A is a fuzzy $\alpha-$open set iff* $u = Int_\alpha u$.

Definition 3.3 *A fuzzy set N in a fts X is called a fuzzy α-Neighborhood of a point x of X iff there exists $U \in F\alpha O(X)$ such that $U \leq N$ and $N(x) = U(X) > 0$.*

The proof of the following theorem is obvious.

Theorem 3.4

- *Any union (resp. intersection) of fuzzy α-open (resp.) sets is a fuzzy α-open (resp. fuzzy α-closed) set.*
- *Any finite intersection (resp. union) of fuzzy α-open (resp. fuzzy α-closed) sets is a fuzzy α-open (resp. fuzzy α-closed) set.*

Theorem 3.5 *If M and N are fuzzy α-Neighborhood of x then $N \cap M$ is also a fuzzy α-Neighborhood of x.*

Proof. Since N and M are fuzzy α-Neighborhood of x there exist $G, H \in F\alpha O(X)$ such that $G \leq N$, $H \leq M$, $N(x) = G(X) > 0$ and $M(x) = H(X) > 0$. Then $M \cap H \in F\alpha O(X)$. ■

Proposition 3.6 *The collection of all fuzzy α-open sets ($F\alpha O(X)$) is a fuzzy Topology on X, which is finer than fts and denoted by $F\tau^{\alpha}(X)$.*

Proof. From Theorem (3.4) and since 0_x and 1_x is also fuzzy α-open set in X then $F\alpha O(X)$ is a fuzzy topology on X. ■

Theorem 3.7 *For any fuzzy subset u of fts X. u is called " fuzzy α-open set" if and only if there exists a fuzzy open set m such that $m \leq u \leq int\ clm$.*

Proof. If u be a fuzzy α-open set $\Rightarrow u \leq int\ cl\ intu$. Hence $m \leq u \leq int\ clm$, where $m = intu$.

Converse is obvious. ■

Theorem 3.8 *For any fuzzy subset u of a fuzzy space X, the following properties are equivalent:*

(i) $v \in F\alpha C(X)$

(ii) $cl\ Int\ cl\ v \leq v$

(iii) *There exists a fuzzy closed set say n such that* $cl\ Intn \leq v \leq n$

Proof.

$(i) \Rightarrow (ii)$. Let $v \in F\alpha C(X)$, then $v^c \in F\alpha O(X)$. Hence $v^c \leq Int\ cl\ Int\ v^c$, then $cl\ Int\ clA \leq\ A$.

$(ii) \Rightarrow (iii)$. Let $cl\ Int\ clv \leq\ v$. Hence $cl\ Int\ clv \leq\ v \leq cl\ v$. Then there exist a fuzzy closed set (clv) such that $cl\ Int\ n \leq\ v \leq n$, where $n = cl\ v$

$(iii) \Rightarrow (i)$. Suppose that there exists a fuzzy closed set say n such that $cl\ Int\ n \leq v \leq n$. It is clear that $n^c \leq v^c \leq Int\ cl\ n^c$.
Therefore $v \in F\alpha C(X)$. ■

Theorem 3.9 *Let X and Y be two topological spaces $u \in F\tau^{\alpha}(X)$ and $v \in F\tau^{\alpha}(Y)$, then $u \times v \in F\tau^{\alpha}(X \times Y)$.*

Proof. Since $u \leq Int\ cl\ Int\ u, v \leq Int\ cl\ Int\ v$. Hence $(u \times v) \leq Int\ cl\ Int\ (u \times v)$. Therefore $u \times v \in F\tau^{\alpha}(X \times Y)$. ■

Lemma 3.10 [7] *Let u be a fuzzy open set in a fts, then $Intclu = Sclu$.*

From the following Theorem, we can see fuzzy α-open set **equal** fuzzy feebly open set.

Theorem 3.11 *Let u fuzzy subset of a $fts(X)$, then u is feebly open set iff $u \in F\tau^{\alpha}(X)$*

Proof. this immediate consequence of Definition (2.4) and Lemma (3.10). ■

From Definition (2.4), Theorem (3.7) and Lemma (3.10), easily imply the following theorem.

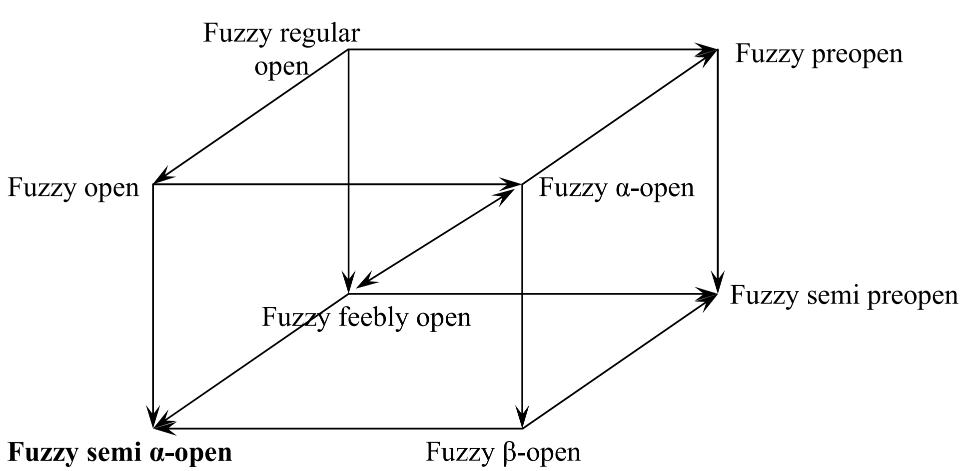
Theorem 3.12 *For any fuzzy subset of a fts X, the following properties are equivalent:*

(i) $u \in F\tau^{\alpha}(X)$
(ii) $m \leq u \leq Int\ clm$*, for some fuzzy open sets* m.
(iii) $m \leq u \leq Sclm$*, for some fuzzy open sets* m.
(v) $u \leq Scl\ (Intu)$

Proposition 3.13 *Let u and v fuzzy subsets of a $fts(X)$, $u \in F\tau^{\alpha}(X)$ and $u \leq v \leq Int\ clu$, then $v \in F\tau^{\alpha}(X)$.*

Proof. Since $u \in F\tau^{\alpha}(X)$, $v \leq Int\ clu \leq Int\ cl\ (Int\ cl\ Intu) \leq nt\ cl\ Intv$. This show that $v \in F\tau^{\alpha}(X)$. ■

The following Diagram (1) [6] explains the relationship between different classes of weakly open sets



4 Fuzzy α-continuous mapping and fuzzy α-open (α-closed)mapping

In this section, we will use the concepts of α-open and semi α-open set to define some new weakly types of α-continuous (α-open) mappings. Also, we will prove some theorems, properties about these concepts.

Definition 4.1 *A mapping $f : X \to Y$ is said to be:*

- *Fuzzy continuous [3] if $f^{-1}(v)$ is fuzzy open (fuzzy closed) set in X for each fuzzy open (fuzzy closed) set v in Y.*
- *Fuzzy $\alpha-$continuous [8] if $f^{-1}(v)$ is fuzzy $\alpha-$open (fuzzy $\alpha-$closed) set in X for each fuzzy open (fuzzy closed) set u in Y.*
- *Fuzzy semi $\alpha-$continuous [6] if $f^{-1}(v)$ is a fuzzy semi $\alpha-$open (fuzzy semi $\alpha-$closed) set in X for each fuzzy open (fuzzy closed) set v in Y.*
- *Fuzzy semi $\alpha^{\star}-$continuous [6] if $f^{-1}(v)$ is a fuzzy semi $\alpha-$open (fuzzy semi $\alpha-$closed) set in X for each fuzzy semi $\alpha-$open (fuzzy semi $\alpha-$closed) set v in Y.*
- *Fuzzy semi $\alpha^{\star\star}-$continuous [6] if $f^{-1}(v)$ is a fuzzy open (closed) set in X for each fuzzy semi $\alpha-$open (fuzzy semi $\alpha-$closed) set v in Y.*
- *Fuzzy semi α-open [6] if $f(u)$ is fuzzy semi α-open (semi α-closed) set in Y for each fuzzy open (closed) set u in X.*
- *Fuzzy semi $\alpha^{\star}$-open [6] if $f(u)$ is fuzzy semi α-open (semi α-closed) set in Y for each fuzzy semi α-open (semi α-closed) set u in X.*
- *Fuzzy semi $\alpha^{\star\star}$-open [6] if $f(u)$ is fuzzy open (closed) set in Y for each fuzzy semi α-open (semi α-closed) set u in X*

Definition 4.2 *A mapping $f : X \to Y$ is said to be:*

- *Fuzzy $\alpha^{\star}$-continuous if $f^{-1}(v)$ is fuzzy α-open (fuzzy α-closed) set in X for each fuzzy α-open (fuzzy α-closed) set v in Y.*
- *Fuzzy $\alpha^{\star\star}$-continuous if $f^{-1}(v)$ is fuzzy open (fuzzy closed) set in X for each fuzzy α-open (fuzzy α-closed) set v in Y.*

Definition 4.3 *A mapping $f : X \to Y$ is said to be:*

- *Fuzzy α-open if $f(u)$ is fuzzy α-open (α-closed) set in Y for each fuzzy open (closed) set u in X.*

- *Fuzzy $\alpha^\star$-open if $f(u)$ is fuzzy α-open (α-closed) set in Y for each fuzzy α-open (α-closed) set u in X.*
- *Fuzzy $\alpha^{\star\star}$-open if $f(u)$ is fuzzy open (closed) set in Y for each fuzzy α-open (α-closed) set u in X.*

Theorem 4.4 *If $f : (X, \tau) \to (Y, \sigma)$ is a fuzzy open and fuzzy semi α-continuous mapping, then f is fuzzy semi $\alpha^\star$-continuous.*

Proof. For any arbitrary $v \in FS\alpha O(Y)$, there exists a fuzzy α-open set n in Y such that $n \leq v \leq cln$.
Since f is fuzzy open, we have $f^{-1}(n) \leq f^{-1}(v) \leq clf^{-1}(n)$ and f is also fuzzy semi α-continuous and n is open in Y, $f^{-1}(n) \in FS\alpha O(X)$.
We obtain $f^{-1}(v) \in FS\alpha O(X)$, then f is fuzzy semi $\alpha^\star$-continuous. ■

Proposition 4.5

(i) If $f : (X, \tau) \to (Y, \sigma)$ is a fuzzy open, fuzzy continuous and bijective, then f is fuzzy $\alpha^\star$-continuous.

(ii) If $f : (X, \tau) \to (Y, \sigma)$ is a fuzzy $\alpha^\star$-continuous iff $f : (X, \tau^\alpha) \to (Y, \sigma^\alpha)$ is fuzzy continuous.

Proof. We only prove (1) and the other is easy to prove it.
Let $v \in \tau^\alpha(X)$ and let fuzzy singleton $p \in f^{-1}(v) \Rightarrow f(p) \in v$. Since $v \in \tau^\alpha(Y)$ therefore, there exists a fuzzy open set n in Y such that $v \leq n \leq clIntv$, $f(p) \in n \leq clIntv$, then $p \in f^{-1}(n) \leq cl\ Intf^{-1}(v)$.
Therefore, $p \in Intcl\ Intf^{-1}(v)$. Hence $f^{-1}(v) \leq Int\ cl\ Intf^{-1}(v)$.
Then f is fuzzy $\alpha^\star$-continuous. ■

Corollary 4.6

(i) If $f : (X, \tau) \to (Y, \sigma)$ is a fuzzy open and fuzzy continuous, then f is fuzzy $\alpha^\star$-open mapping.

(ii) If $f : (X, \tau) \to (Y, \sigma)$ is a fuzzy $\alpha^\star$-open iff $f : (X, \tau^\alpha) \to (Y, \sigma^\alpha)$ is fuzzy open.

Proof. It is analogous proposition(4.5). ■

The following diagram explains the relationship between different weakly classes of α- continuous mappings

Diagram(2)

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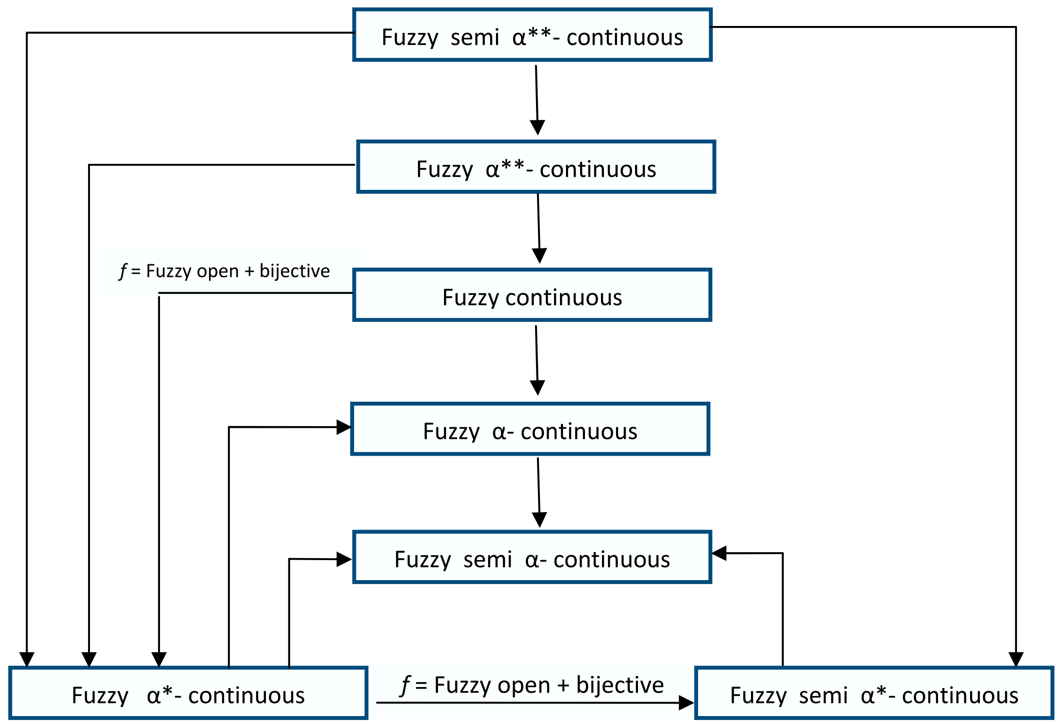
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f = Fuzzy open + bijective
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