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Computation of Topological Indices of Mesh, Grid, Torus and Cylinder

R. Pradeep Kumar[1], **Soner Nandappa D.**[2]
and M. R. Rajesh Kanna[3]

[1] University of Mysore, Mysuru 570 006, India

[2] Department of Mathematics
University of Mysore, Mysuru - 570 006, India

[3] Department of Mathematics
Maharani's Science College for Women
J. L. B. Road, Mysuru - 570 005, India
&
Governement First Grade College
Bettampady, Puttur, D.K, India

Abstract

In this paper, we compute *ABC* index, Randic connectivity index, Sum connectivity index and *GA* index of grid, extended grid, torus and cylinder.

Mathematics Subject Classification: 05C12, 05C90

Keywords: *ABC* index, Randic connectivity index, Sum connectivity index, *GA* index, Grid, Extended grid, Torus and Cylinder

1 Introduction

Topological indices are the molecular descriptors that describes the structures of chemical compounds and it help us to predict certain physico-chemical properties like boiling point, enthalpy of vaporization, stability etc. In this paper, we determine the topological indices like atom-bond connectivity index, sum connectivity index, randic connectivity index and geometric-arithmetic connectivity index of mesh, grid, torus and cylinder.

All molecular graphs considered in this paper are finite, connected, loop less and without multiple edges. Let $G = (V, E)$ be a graph with n vertices and m edges. The degree of a vertex $u \in V(G)$ is denoted by d_u and is the number of vertices that are adjacent to u. The edge connecting the vertices u and v is denoted by uv. Using these terminologies, certain topological indices are defined in the following manner.

The Atom-bond connectivity index, ABC index is one of the degree based molecular descriptor, which was introduced by Estrada et al. [5] in late 1990's and it can be used for modeling thermodynamic properties of organic chemical compounds, it is also used as a tool for explaining the stability of branched alkanes [6]. Some upper bounds for the atom-bond connectivity index of graphs can be found in [2], The atom-bond connectivity index of chemical bicyclic graphs, connected graphs can be seen in [3, 16]. For further results on ABC index of trees see the papers [9, 11, 15, 17] and the references cited there in.

Definition 1.1. *Let $G = (V, E)$ be a molecular graph and d_u is the degree of the vertex u, then ABC index of G is defined as,* $ABC(G) = \sum_{uv \in E} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$.

The first and oldest degree based topological index is Randic index [13] denoted by $\chi(G)$ and was introduced by Milan Randic in 1975. It provides a quantitative assessment of branching of molecules.

Definition 1.2. *For the graph G Randic index is defined as,* $\chi(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}$.

Sum connectivity index belongs to a family of Randic like indices and it was introduced by Zhou and Trinajstic [19]. Further studies on Sum connectivity index can be found in [20, 21].

Definition 1.3. *For a simple connected graph G, its sum connectivity index $S(G)$ is defined as,* $S(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u + d_v}}$.
.

The Geometric-arithmetic index, $GA(G)$ index of a graph G was introduced by D. Vukicevic et.al[14]. Further studies on GA index can be found in [1, 4, 18]

Definition 1.4. *Let G be a graph and $e = uv$ be an edge of G then Geometric-arithmetic index is defined as,* $GA(G) = \sum_{e=uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u d_v}$.

2 Main results

Theorem 2.1. *Atom bond connectivity index of grid with $(m-1)$ rows and $(n-1)$ columns is given by* $ABC(G(m,n)) =$

$$\begin{cases} \frac{(8\sqrt{6}+6\sqrt{10}-45)m+(8\sqrt{6}+6\sqrt{10}-45)n+18mn+(108-24\sqrt{10}+48\sqrt{3}-48\sqrt{6})}{6\sqrt{6}} & if\ m>2\ and\ n>2 \\ \frac{18-16\sqrt{2}+6\sqrt{2}m}{3\sqrt{2}} & if\ m=2\ and\ n>2 \\ 2\sqrt{2} & if\ m=n=2 \end{cases}$$

Proof. The topological structure of a grid network, denoted by $G(m,n)$, is defined as the cartesian product $P_m \times P_n$ of undirected paths P_m and P_n. The spectrum of the graph does not depend on the numbering of the vertices. However, here we adopt a particular numbering such that the edges has a pattern which is common for any dimension. We follow the sequential numbering from left to right as shown in the diagram.

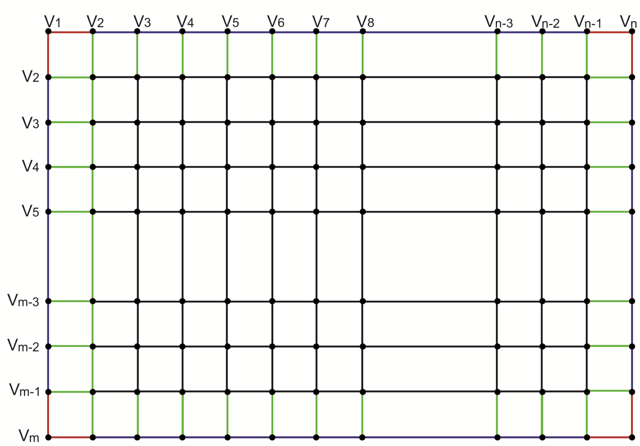

Figure 1

Consider a two-dimensional structure of Grid with $(m-1)$ rows and $(n-1)$ columns as shown in the Figure-1. Let $|e_{i,j}|$ denotes the number of edges connecting the vertices of degrees d_i and d_j.

Case 1 : If $m>2$ and $n>2$, Grid contains only $e_{2,3}$, $e_{3,3}$, $e_{3,4}$ and $e_{4,4}$ edges. In the above figure $e_{2,3}$, $e_{3,3}$, $e_{3,4}$ and $e_{4,4}$ edges are colored in red, blue, green and black respectively. The number of $e_{2,3}$, $e_{3,3}$, $e_{3,4}$ and $e_{4,4}$ edges in each row is mentioned in the following table.

Row	$\|e_{2,3}\|$	$\|e_{3,3}\|$	$\|e_{3,4}\|$	$\|e_{4,4}\|$
1	4	$n-3$	n	$n-3$
2	0	2	2	$2n-5$
3	0	2	2	$2n-5$
4	0	2	2	$2n-5$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$m-2$	0	2	2	$2n-5$
$m-1$	4	$n-3$	$n-2$	0
Total	8	$2m+2n-12$	$2m+2n-8$	$2mn-5m-5n+12$

$\therefore |e_{2,3}| = 8$, $|e_{2,3}| = (2m+2n-12)$, $|e_{3,4}| = (2m+2n-8)$ and $|e_{4,4}| = (2mn-5m-5n+12)$.

Consider, $ABC(G(m,n)) = \sum_{uv \in E(G(m,n))} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$

$= |e_{2,3}|\sqrt{\frac{2+3-2}{2\cdot 3}} + |e_{3,3}|\sqrt{\frac{3+3-2}{3\cdot 3}} + |e_{3,4}|\sqrt{\frac{3+4-2}{3\cdot 4}} + |e_{4,4}|\sqrt{\frac{4+4-2}{4\cdot 4}}$

$= 8\sqrt{\frac{2+3-2}{2\cdot 3}} + 2(m+n-6)\sqrt{\frac{3+3-2}{3\cdot 3}} + 2(m+n-4)\sqrt{\frac{3+4-2}{3\cdot 4}} +$
$(2mn-5m-5n+12)\sqrt{\frac{4+4-2}{4\cdot 4}}$

$= \frac{(8\sqrt{6}+6\sqrt{10}-45)m+(8\sqrt{6}+6\sqrt{10}-45)n+18mn+(108-24\sqrt{10}+48\sqrt{3}-48\sqrt{6})}{6\sqrt{6}}.$

Case 2 : If $m = 2$ and $n > 2$

In this case Grid contains $e_{2,2}$, $e_{2,3}$ and $e_{3,3}$ edges. The edges $e_{2,2}$, $e_{2,3}$ and $e_{3,3}$ are colored in red, blue and black respectively as shown in the Figure 2. The number of $e_{2,2}$, $e_{2,3}$ and $e_{3,3}$ edges in each row is mentioned in the following table.

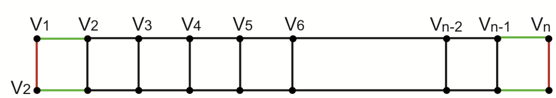

Figure 2

$\|e_{2,2}\|$	$\|e_{2,3}\|$	$\|e_{3,3}\|$
2	4	$(3n-8)$

$$ABC(G(2,n)) = \sum_{uv \in E(G(2,n))} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

$= |e_{2,2}|\sqrt{\frac{2+2-2}{2\cdot 2}} + |e_{2,3}|\sqrt{\frac{2+3-2}{2\cdot 3}} + |e_{3,3}|\sqrt{\frac{3+3-2}{3\cdot 3}}$

$= 2\sqrt{\frac{2+2-2}{2\cdot 2}} + 4\sqrt{\frac{2+3-2}{2\cdot 3}} + (3n-8)\sqrt{\frac{3+3-2}{3\cdot 3}}$

$$= \frac{18 - 16\sqrt{2} + 6\sqrt{2}n}{3\sqrt{2}}$$

Similarly, $ABC(G(m, 2)) = \dfrac{18 - 16\sqrt{2} + 6\sqrt{2}m}{3\sqrt{2}}$

Case 3 : In this case the number of $e_{2,2}$ edges is as shown in Figure 3.

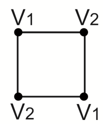

Figure 3

$$ABC(G(2, 2)) = \sum_{uv \in E(G(2,2))} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}} = |e_{2,2}|\sqrt{\frac{2+2-2}{2 \cdot 2}} = 4\sqrt{\frac{2+2-2}{2 \cdot 2}} = 2\sqrt{2}. \quad \square$$

Theorem 2.2. *Randic index of grid with* $(m-1)$ *rows and* $(n-1)$ *columns is given by* $\chi(G(m, n))$ $=$

$$\begin{cases} \frac{(12\sqrt{2}-7\sqrt{6})m+(12\sqrt{2}-7\sqrt{6})n+6\sqrt{6}mn+(96-12\sqrt{6}-48\sqrt{2})}{12\sqrt{6}} & if\ m > 2\ and\ n > 2 \\ \frac{3\sqrt{6}n+(12-5\sqrt{6})}{3\sqrt{6}} & if\ m = 2\ and\ n > 2 \\ 2 & if\ m = n = 2 \end{cases}$$

Proof. **Case 1 :** If $m > 2$ and $n > 2$

Consider, $\chi(G(m, n)) = \displaystyle\sum_{uv \in E(\chi(G(m,n)))} \frac{1}{\sqrt{d_u d_v}}$

$= |e_{2,3}|\frac{1}{\sqrt{2 \cdot 3}} + |e_{3,3}|\frac{1}{\sqrt{3 \cdot 3}} + |e_{3,4}|\frac{1}{\sqrt{3 \cdot 4}} + |e_{4,4}|\frac{1}{\sqrt{4 \cdot 4}}$

$= 8\frac{1}{\sqrt{2 \cdot 3}} + 2(m + n - 6)\frac{1}{\sqrt{3 \cdot 3}} + 2(m + n - 4)\frac{1}{\sqrt{3 \cdot 4}} + (2mn - 5m + 5n + 12)\frac{1}{\sqrt{4 \cdot 4}}$

$= \frac{(12\sqrt{2}-7\sqrt{6})m+(12\sqrt{2}-7\sqrt{6})n+6\sqrt{6}mn+(96-12\sqrt{6}-48\sqrt{2})}{12\sqrt{6}}$

Case 2 : If $m = 2$ and $n > 2$

Consider, $\chi(G(2, n)) = \displaystyle\sum_{uv \in E(\chi(G(2,n)))} \frac{1}{\sqrt{d_u d_v}}$

$= |e_{2,2}|\frac{1}{\sqrt{2 \cdot 2}} + |e_{2,3}|\frac{1}{\sqrt{2 \cdot 3}} + |e_{3,3}|\frac{1}{\sqrt{3 \cdot 3}}$

$= 2\frac{1}{\sqrt{2 \cdot 2}} + 4\frac{1}{\sqrt{2 \cdot 3}} + (3n - 8)\frac{1}{\sqrt{3 \cdot 3}}$

$= \frac{3\sqrt{6}n+(12-5\sqrt{6})}{3\sqrt{6}}$

Similarly, $\chi(G(m, 2)) = \frac{3\sqrt{6}m+(12-5\sqrt{6})}{3\sqrt{6}}$

Case 3 : If $m = 2$ and $n = 2$

Consider, $\chi(G(2, 2)) = \displaystyle\sum_{uv \in E(\chi(G(2,2)))} \frac{1}{\sqrt{d_u d_v}} = |e_{2,2}|\frac{1}{\sqrt{2 \cdot 2}} = 4\frac{1}{\sqrt{4}} = 2$ $\square$

Theorem 2.3. *Sum Connectivity index of Grid with* $(m-1)$ *rows and* $(n-1)$ *columns in each row is given by* $S(G(m, n))$

$$= \begin{cases} \frac{(4\sqrt{35}+4\sqrt{30}-5\sqrt{105})m+(4\sqrt{35}+4\sqrt{30}-5\sqrt{105})n+2\sqrt{105}mn+(16\sqrt{42}-24\sqrt{35}-16\sqrt{30}+12\sqrt{105})}{2\sqrt{210}} & if\ m>2\ and\ n>2 \\ \frac{3\sqrt{5}n+(\sqrt{30}+4\sqrt{6}-8\sqrt{5})}{\sqrt{30}} & if\ m=2\ and\ n>2 \\ 2 & if\ m=n=2 \end{cases}$$

Proof. **Case 1 :** If $m>2$ and $n>2$

Sum Connectivity Index is given by $S(G(m,n)) = \sum_{uv\in E(G(m,n))} \frac{1}{\sqrt{d_u+d_v}}$.

$= |e_{2,3}|\frac{1}{\sqrt{2+3}} + |e_{3,3}|\frac{1}{\sqrt{3+3}} + |e_{3,4}|\frac{1}{\sqrt{3+4}} + |e_{4,4|}\frac{1}{\sqrt{4+4}}$

$= 8\frac{1}{\sqrt{2+3}} + 2(m+n-6)\frac{1}{\sqrt{3+3}} + 2(m+n-4)\frac{1}{\sqrt{3+4}} + (2mn-5m+5n+12)\frac{1}{\sqrt{4+4}}$

$= \frac{(4\sqrt{35}+4\sqrt{30}-5\sqrt{105})m+(4\sqrt{35}+4\sqrt{30}-5\sqrt{105})n+2\sqrt{105}mn+(16\sqrt{42}-24\sqrt{35}-16\sqrt{30}+12\sqrt{105})}{2\sqrt{210}}$

Case 2 : If $m=2$ and $n>2$

Consider, $S(G(2,n)) = \sum_{uv\in E(G(2,n))} \frac{1}{\sqrt{d_u+d_v}}$.

$= |e_{2,2}|\frac{1}{\sqrt{2+2}} + |e_{2,3}|\frac{1}{\sqrt{2+3}} + |e_{3,3}|\frac{1}{\sqrt{3+3}}$

$= 2\frac{1}{\sqrt{2+2}} + 4\frac{1}{\sqrt{2+3}} + (3n-8)\frac{1}{\sqrt{3+3}}$

$= \frac{3\sqrt{5}n+(\sqrt{30}+4\sqrt{6}-8\sqrt{5})}{\sqrt{30}}$

Similarly, $S(G(m,2)) = \frac{3\sqrt{5}m+(\sqrt{30}+4\sqrt{6}-8\sqrt{5})}{\sqrt{30}}$.

Case 3 : If $m=2$ and $n=2$

Consider, $S(G(2,2)) = \sum_{uv\in E(\chi(G(2,2)))} \frac{1}{\sqrt{d_u d_v}} = |e_{2,2}|\frac{1}{\sqrt{2\cdot 2}} = 4\frac{1}{\sqrt{4}} = 2$ □

Theorem 2.4. *Geometric-arithmetic(GA) index of Grid with $(m-1)$ rows and $(n-1)$ columns in each row is given by $GA(G(m,n))$*

$$= \begin{cases} \frac{(40\sqrt{3}-105)m+(40\sqrt{3}-105)n+70mn+(102\sqrt{6}-160\sqrt{3}-840)}{2\sqrt{210}} & if\ m>2\ and\ n>2 \\ \frac{15n+(8\sqrt{6}-30)}{5} & if\ m=2\ and\ n>2 \\ 4 & if\ m=n=2 \end{cases}$$

Proof. **Case 1 :** If $m>2$ and $n>2$

Geometric-Arithmetic Index is given by $GA(G(m,n)) = \sum_{e=uv\in E(G(m,n))} \frac{2\sqrt{d_u d_v}}{d_u d_v}$

$= |e_{2,3}|\frac{2\sqrt{2\cdot 3}}{2+3} + |e_{3,3}|\frac{2\sqrt{3\cdot 3}}{3+3} + |e_{3,4}|\frac{2\sqrt{3\cdot 4}}{3+4} + |e_{4,4}|\frac{2\sqrt{4\cdot 4}}{4+4}$

$= 8\frac{2\sqrt{6}}{5} + 2(m+n-6)\frac{6}{6} + 2(m+n-4)\frac{4\sqrt{3}}{7} + (2mn-5m-5n+12)\frac{8}{8}$

$= \frac{(40\sqrt{3}-105)m+(40\sqrt{3}-105)n+70mn+(102\sqrt{6}-160\sqrt{3}-840)}{2\sqrt{210}}$

Case 2 : If $m=2$ and $n>2$

Consider, $GA(G(2,n)) = \sum_{uv\in E(G(2,n))} \frac{2\sqrt{d_u d_v}}{d_u d_v}$.

$= |e_{2,2}|\frac{2\sqrt{2\cdot 2}}{2+2} + |e_{2,3}|\frac{2\sqrt{2\cdot 3}}{2+3} + |e_{3,3}|\frac{2\sqrt{3\cdot 3}}{3+3}$
$= 2\frac{2\sqrt{2\cdot 2}}{2+2} + 4\frac{2\sqrt{2\cdot 3}}{2+3} + (3n-8)\frac{2\sqrt{3\cdot 3}}{3+3}$
$= \frac{15n+(8\sqrt{6}-30)}{5}$.

Similarly, $GA(G(m,2)) = \frac{15m+(8\sqrt{6}-30)}{5}$.

Case 3 : If $m = 2$ and $n = 2$

Consider, $GA(G(2,2)) = \sum_{uv\in E(\chi(G(2,2)))} \frac{2\sqrt{d_u d_v}}{d_u d_v} = |e_{2,2}|\frac{2\sqrt{2\cdot 2}}{2+2} = 4\frac{2\sqrt{2\cdot 2}}{2+2} = 4$ □

Theorem 2.5. *Atom bond connectivity index of Extended Grid with $(m-1)$ rows and $(n-1)$ columns in each row is given by* $ABC(EX(m,n)) =$

$$\begin{cases} \frac{(32+12\sqrt{55}-55\sqrt{7})m+(32+12\sqrt{55}-55\sqrt{7})n+20\sqrt{7}mn+(64\sqrt{5}+40\sqrt{3}-128-150\sqrt{7})}{20\sqrt{2}} & if \quad m>2\, and\, n>2 \\ \frac{30\sqrt{2}n+(20+24\sqrt{10}-84\sqrt{2})}{15} & if \quad m=2\, and\, n>2 \\ 4 & if \quad m\, and\, n=2 \end{cases}$$

Proof. By making each 4-cycle in a $m \times n$ mesh into a complete graph we obtain an architecture called an extended mesh denoted by $EX(m,n)$. The number of vertices in $EX(m,n)$ is mn and the number of edges is $4mn + 3m + 3n + 2$. We follow the sequential numbering from left to right.

Consider a extended grid with $(m-1)$ rows and $(n-1)$ columns. Let $e_{i,j}$ denotes the number of edges connecting the vertices of degrees d_i and d_j. Two-dimensional structure of extended grid as shown in the Figure 4 and it contains $e_{3,5}$, $e_{3,8}$, $e_{5,5}$, $e_{5,8}$ and $e_{8,8}$ edges. In the above figure $e_{3,5}$, $e_{3,8}$, $e_{5,5}$, $e_{5,8}$ and $e_{8,8}$ edges are colored in red, purple, green, yellow and black respectively. The number of edges of these types in each row is mentioned in the following table - 3.

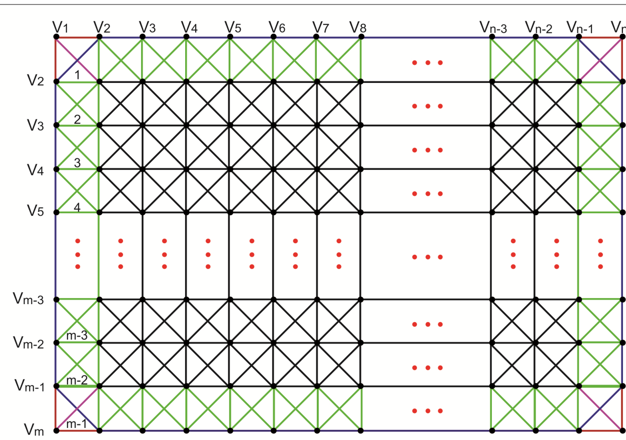

Figure 4

Row	$\lvert e_{3,5}\rvert$	$\lvert e_{3,8}\rvert$	$\lvert e_{5,5}\rvert$	$\lvert e_{5,8}\rvert$	$\lvert e_{8,8}\rvert$
1	4	2	n-1	$3n-6$	$n-3$
2	0	0	2	6	$4n-11$
3	0	0	2	6	$4n-11$
4	0	0	2	6	$4n-11$
. . .	. . .	. . .	. . .	. . .	. . .
$m-2$	0	0	2	6	$4n-11$
$m-1$	4	2	$n-1$	$3n-8$	0
Total	8	4	$2m+2n-8$	$6m+6n-32$	$4mn-11m-11n+30$

$\therefore |e_{3,5}| = 8$, $|e_{3,8}| = 2$, $|e_{5,5}| = (2m+2n-8)$, $|e_{5,8}| = (6m+6n-32)$ and $|e_{8,8}| = (4mn-11m-11n+30)$.

Case 1 : The atom-bond connectivity index of extended grid for $m > 3$, $n > 3$ is

Consider, $ABC(EX(m,n)) = \sum_{uv\in E(EX(m,n))} \sqrt{\frac{d_u+d_v-2}{d_u d_v}}$

$$= |e_{3,5}|\sqrt{\frac{3+5-2}{3\cdot 5}} + |e_{3,8}|\sqrt{\frac{3+8-2}{3\cdot 8}} + |e_{5,5}|\sqrt{\frac{5+5-2}{5\cdot 5}} + |e_{5,8}|\sqrt{\frac{5+8-2}{5\cdot 8}} + |e_{8,8}|\sqrt{\frac{8+8-2}{8\cdot 8}}$$

$$= 8\sqrt{\frac{3+5-2}{3\cdot 5}} + 4\sqrt{\frac{3+8-2}{3\cdot 8}} + 2(m+n-4)\sqrt{\frac{5+5-2}{5\cdot 5}} + 2(3m+3n-16)\sqrt{\frac{5+8-2}{5\cdot 8}}$$

$$+(4mn-11m-11n+30)\sqrt{\frac{8+8-2}{8\cdot 8}}$$

$$= \frac{(32+12\sqrt{55}-55\sqrt{7})m+(32+12\sqrt{55}-55\sqrt{7})n+20\sqrt{7}mn+(64\sqrt{5}+40\sqrt{3}-128-80\sqrt{55}+150\sqrt{7})}{20\sqrt{2}}$$

Case 2 : If $m = 2$ and $n > 2$

In this case extended grid contains $e_{3,3}$, $e_{3,5}$ and $e_{5,5}$ edges. The edges $e_{3,3}$, $e_{3,5}$ and $e_{5,5}$ are colored in red, blue and black respectively as shown in the Figure 5. The number of $e_{3,3}$, $e_{3,5}$ and $e_{5,5}$ edges in each row is mentioned in the following table.

$\lvert e_{3,3}\rvert$	$\lvert e_{3,5}\rvert$	$\lvert e_{5,5}\rvert$
2	8	$(5n-14)$

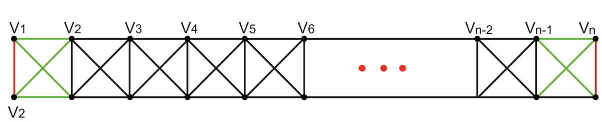

Figure 5

$ABC(EX(2,n)) = \sum_{uv\in E(EX(2,n))} \sqrt{\frac{d_u+d_v-2}{d_u d_v}}$

$= |e_{3,3}|\sqrt{\dfrac{3+3-2}{3\cdot 3}} + |e_{3,5}|\sqrt{\dfrac{3+5-2}{3\cdot 5}} + |e_{5,5}|\sqrt{\dfrac{5+5-2}{5\cdot 5}}$
$= \frac{30\sqrt{2}n+(20+24\sqrt{10}-84\sqrt{2})}{15}$

Similarly, $ABC(EX(m,2)) = \frac{30\sqrt{2}m+(20+24\sqrt{10}-84\sqrt{2})}{15}$

Case 3 : If $m = 2$ and $n = 2$

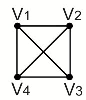

Figure 6

Consider, $ABC(EX(2,2)) = |e_{3,3}|\sqrt{\dfrac{3+3-2}{3\cdot 3}} = 6\sqrt{\dfrac{3+3-2}{3\cdot 3}} = 4.$ □

Theorem 2.6. *The Randic index of extended grid with $(m-1)$ rows and $(n-1)$ columns in each row is given by* $\chi(EX(m,n))$

$$= \begin{cases} \frac{(12\sqrt{30}-39\sqrt{3})m+(12\sqrt{30}-39\sqrt{3})n+20\sqrt{3}mn+(64\sqrt{5}+40\sqrt{2}-64\sqrt{30}+86\sqrt{3})}{40\sqrt{3}} & if \quad m>2\, and n>2 \\ \frac{15n+(8\sqrt{15}-32)}{15} & if \quad m=2\, and n>2 \\ 2 & if \quad m=2\, and n=2 \end{cases}$$

Proof. **Case 1 :** If $m > 2$ and $n > 2$
Randic index of extended grid is $\chi(EX(m,n)) = \sum_{uv\in E(\chi(EX(m,n)))} \dfrac{1}{\sqrt{d_u d_v}}$
$= |e_{3,5}|\dfrac{1}{\sqrt{3\cdot 5}} + |e_{3,8}|\dfrac{1}{\sqrt{3\cdot 8}} + |e_{5,5}|\dfrac{1}{\sqrt{5\cdot 5}} + |e_{5,8}|\dfrac{1}{\sqrt{5\cdot 8}} + |e_{8,8}|\dfrac{1}{\sqrt{8\cdot 8}}$
$= 8\dfrac{1}{\sqrt{3\cdot 5}} + 4\dfrac{1}{\sqrt{3\cdot 8}} + 2(m+n-4)\dfrac{1}{\sqrt{5\cdot 5}} + (6m+6n-32)\dfrac{1}{\sqrt{5\cdot 8}} + (4mn-11m-11n+30)\dfrac{1}{\sqrt{8\cdot 8}}$
$= \frac{(12\sqrt{30}-39\sqrt{3})m+(12\sqrt{30}-39\sqrt{3})n+20\sqrt{3}mn+(64\sqrt{5}+40\sqrt{2}-64\sqrt{30}+86\sqrt{3})}{40\sqrt{3}}$

Case 2 : If $m = 2$ and $n > 2$
Consider, $\chi(EX(2,n)) = \sum_{uv\in E(\chi(EX(2,n)))} \dfrac{1}{\sqrt{d_u d_v}}$
$= |e_{3,3}|\dfrac{1}{\sqrt{3\cdot 3}} + |e_{3,5}|\dfrac{1}{\sqrt{3\cdot 5}} + |e_{5,5}|\dfrac{1}{\sqrt{5\cdot 5}}$
$= 2\dfrac{1}{\sqrt{3\cdot 3}} + 8\dfrac{1}{\sqrt{3\cdot 5}} + (5n-14)\dfrac{1}{\sqrt{5\cdot 5}}$
$= \frac{15n+(8\sqrt{15}-32)}{15}$
Similarly, $\chi(EX(m,2)) = \frac{15m+(8\sqrt{15}-32)}{15}$

Case 3 : If $m = 2$ and $n = 2$
Consider, $\chi(EX(2,2)) = \sum_{uv\in E(\chi(EX(2,2)))} \dfrac{1}{\sqrt{d_u d_v}} = |e_{3,3}|\frac{1}{\sqrt{3\cdot 3}} = \frac{6}{3} = 2$ □

Theorem 2.7. *Sum Connectivity index of Extended Grid with $(m-1)$ rows and $(n-1)$ columns in each row is given by* $S(EX(m,n)) =$

$$\begin{cases} \frac{(8\sqrt{143}+24\sqrt{110}-11\sqrt{1430})m+(8\sqrt{143}+24\sqrt{110}-11\sqrt{1430})n+4\sqrt{1430}mn+(8\sqrt{2860}+16\sqrt{130}-128\sqrt{110}+30\sqrt{1430})}{4\sqrt{1430}} & if \quad m>2\, and\, n>2 \\ \frac{5\sqrt{3}n+(2\sqrt{5}+4\sqrt{15}-14\sqrt{3})}{\sqrt{30}} & if \quad m\, or\, n=2 \\ \sqrt{6} & if \quad m\, and\, n=2 \end{cases}$$

Proof. **Case 1 :** If $m>2$ and $n>2$

Sum Connectivity Index of extended grid is given by $S(EX(m,n)) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u+d_v}}$

$$= |e_{3,5}|\frac{1}{\sqrt{3+5}} + |e_{3,8}|\frac{1}{\sqrt{3+8}} + |e_{5,5}|\frac{1}{\sqrt{5+5}} + |e_{5,8}|\frac{1}{\sqrt{5+8}} + |e_{8,8}|\frac{1}{\sqrt{8+8}}$$

$$= 8\frac{1}{\sqrt{3+5}} + 4\frac{1}{\sqrt{3+8}} + 2(m+n-4)\frac{1}{\sqrt{5+5}} + (6m+6n-32)\frac{1}{\sqrt{5+8}} +(4mn-11m-11n+30)\frac{1}{\sqrt{8+8}}$$

$$= \frac{(8\sqrt{143}+24\sqrt{110}-11\sqrt{1430})m+(8\sqrt{143}+24\sqrt{110}-11\sqrt{1430})n+4\sqrt{1430}mn+(8\sqrt{2860}+16\sqrt{130}-128\sqrt{110}+30\sqrt{1430})}{4\sqrt{1430}}$$

Case 2 : If $m=2$ and $n>2$

Consider, $S(EX(2,n)) = \sum_{uv\in E(EX(G(2,n))} \frac{1}{\sqrt{d_u+d_v}}$

$$= |e_{3,3}|\frac{1}{\sqrt{3+3}} + |e_{3,5}|\frac{1}{\sqrt{3+5}} + |e_{5,5}|\frac{1}{\sqrt{5+5}}$$

$$= 2\frac{1}{\sqrt{3+3}} + 8\frac{1}{\sqrt{3+5}} + (5n-14)\frac{1}{\sqrt{5+5}}$$

$$= \frac{5\sqrt{3}n+(2\sqrt{5}+4\sqrt{15}-14\sqrt{3})}{\sqrt{30}}$$

Similarly, $S(EX(m,2)) = \frac{5\sqrt{3}m+(2\sqrt{5}+4\sqrt{15}-14\sqrt{3})}{\sqrt{30}}$

Case 3 : If $m=2$ and $n=2$

Consider, $S(EX(2,2)) = \sum_{uv\in E(EX(2,2))} \frac{1}{\sqrt{d_u+d_v}} = |e_{3,3}|\frac{1}{\sqrt{3+3}} = 6\frac{1}{\sqrt{6}} = \sqrt{6}.$ □

Theorem 2.8. *Geometric-arithmetic (GA) index of extended grid with $(m-1)$ rows and $(n-1)$ columns in each row is given by* $GA(EX(m,n)) =$

$$\begin{cases} \frac{(264\sqrt{10}-1287)m+(264\sqrt{10}-1287)n+2002mn+(286\sqrt{15}+208\sqrt{6}-1408\sqrt{10}+3146)}{143} & if \quad m>2\, and\, n>2 \\ 5n+(2\sqrt{15}-12) & if \quad m\, or\, n=2 \\ 6 & if \quad m\, and\, n=2 \end{cases}$$

Proof. If $m>2$ and $n>2$

Geometric-Arithmetic Index of extended grid is given by $GA(EX(m,n)) = \sum_{e=uv\in E(EX(m,n))} \frac{2\sqrt{d_u d_v}}{d_u d_v}.$

$$= |e_{3,5}|\frac{2\sqrt{3\cdot 5}}{3+5} + |e_{3,8}|\frac{2\sqrt{3\cdot 8}}{3+8} + |e_{5,5}|\frac{2\sqrt{5\cdot 5}}{5+5} + |e_{5,8}|\frac{2\sqrt{5\cdot 8}}{5+8} + |e_{8,8}|\frac{2\sqrt{8\cdot 8}}{8+8}$$

$$= 8\frac{2\sqrt{3\cdot 5}}{3+5} + 4\frac{2\sqrt{3\cdot 8}}{3+8} + 2(m+n-4)\frac{2\sqrt{5\cdot 5}}{5+5} + 2(3m+3n-16)\frac{2\sqrt{5\cdot 8}}{5+8} +(4mn-11m-11n+30)\frac{2\sqrt{8\cdot 8}}{8+8}$$

$$= \frac{(264\sqrt{10}-1287)m+(264\sqrt{10}-1287)n+2002mn+(286\sqrt{15}+208\sqrt{6}-1408\sqrt{10}+3146)}{143}$$

Case 2 : If $m = 2$ and $n > 2$

Consider, $GA(EX(2,n)) = \sum_{uv \in E(EX(2,n))} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$.

$= |e_{3,3}| \frac{2\sqrt{3 \cdot 3}}{3+3} + |e_{3,5}| \frac{2\sqrt{3 \cdot 5}}{3+5} + |e_{5,5}| \frac{2\sqrt{5 \cdot 5}}{5+5}$

$= 2\frac{2\sqrt{3 \cdot 3}}{3+3} + 8\frac{2\sqrt{3 \cdot 5}}{3+5} + (5n-14)\frac{2\sqrt{5 \cdot 5}}{5+5}$

$= 5n + (2\sqrt{15} - 12)$

Similarly, $GA(G(m,2)) = 5m + (2\sqrt{15} - 12)$

Case 3 : If $m = 2$ and $n = 2$

Consider, $GA(EX(2,2)) = \sum_{uv \in E(EX(2,n))} \frac{2\sqrt{d_u d_v}}{d_u + d_v} = |e_{3,3}| \frac{2\sqrt{3 \cdot 3}}{3+3} = 6$ □

Theorem 2.9. *Atom bond connectivity index of cylinder with* $(m-1)$ *rows and* $(n-1)$ *columns in each row is given by* $ABC(C(m,n)) =$

$$\begin{cases} \frac{(15\sqrt{6}+40\sqrt{3}-40\sqrt{10})m+(48\sqrt{2}-6\sqrt{30}-35\sqrt{10})n+15\sqrt{10}mn+(20\sqrt{15}+138\sqrt{10}-45\sqrt{6}-80\sqrt{3}-144\sqrt{2})}{30} & \text{if } m > 2 \text{ and } n > 2 \\ \frac{30\sqrt{2}n+(20+24\sqrt{10}-84\sqrt{2})}{15} & \text{if } m = 2 \text{ and } n > 2 \\ 4 & \text{if } m = 2 \text{ and } n = 2 \end{cases}$$

Proof. The topological structure of a cylinder network, denoted $C(m,n)$ and is defined as the cartesian product $P_m \times C_n$ of undirected path P_m and an undirected cycle C_n. The numbering of vertices adopted for cylinder is same as that of grid.

Consider a cylinder with $(m-1)$ rows and $(n-1)$ columns. Let $e_{i,j}$ denotes the number of edges connecting the vertices of degrees d_i and d_j. Two-dimensional structure of cylinder is as shown in the Figure 6 and it contains $e_{3,4}$, $e_{3,5}$, $e_{4,4}$, $e_{4,6}$, $e_{5,5}$, $e_{5,6}$ and $e_{6,6}$ edges. In the above figure $e_{3,4}$, $e_{3,5}$, $e_{4,4}$, $e_{4,6}$, $e_{5,5}$, $e_{5,6}$ and $e_{6,6}$ edges are colored in green, blue, red, orange, brown, yellow and black respectively. The number of $e_{3,4}$, $e_{3,5}$, $e_{4,4}$, $e_{4,6}$, $e_{5,5}$, $e_{5,6}$ and $e_{6,6}$ edges in each row is mentioned in the following table - 3.

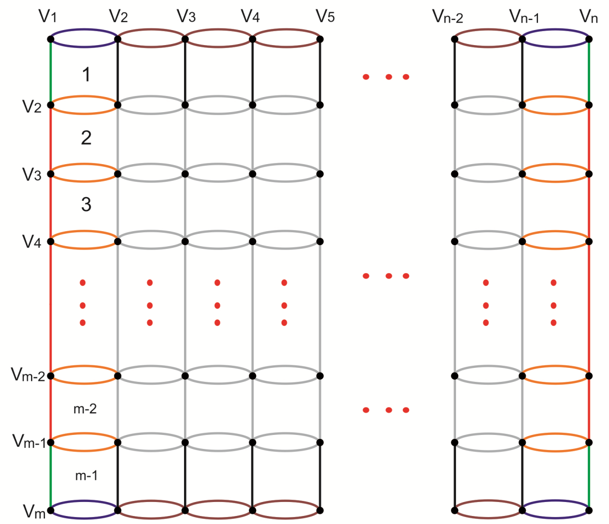

Figure 7

Row	$\|e_{3,4}\|$	$\|e_{3,5}\|$	$\|e_{4,4}\|$	$\|e_{4,6}\|$	$e_{5,5}$	$\|e_{5,6}\|$	$\|e_{6,6}\|$
1	2	4	0	4	$2n-6$	$n-2$	$2n-6$
2	0	0	2	4	0	0	$3n-8$
3	0	0	2	4	0	0	$3n-8$
4	0	0	2	4	0	0	$3n-8$
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
$m-2$	0	0	2	4	0	0	$3n-8$
$m-1$	2	4	0	0	$2n-6$	$n-2$	0
Total	4	8	$2m-6$	$4m-8$	$4n-12$	$2n-4$	$3mn-8m-7n+18$

$\therefore |e_{3,4}| = 4$, $|e_{3,5}| = 8$, $|e_{4,4}| = (2m-6)$, $|e_{4,6}| = (4m-8)$, $|e_{5,5}| = (4n-12)$, $|e_{5,6}| = (2n-4)$ and $|e_{6,6}| = (3mn-8m-7n+18)$.

Case 1 : The atom-bond connectivity index of cylinder for $m > 2$, $n > 2$ is

Consider, $ABC(C(m,n)) = \sum_{uv\in E(C(m,n))} \sqrt{\frac{d_u+d_v-2}{d_ud_v}}$

$$ABC(C(m,n)) = |e_{3,4}|\sqrt{\frac{3+4-2}{3\cdot 4}} + |e_{3,5}|\sqrt{\frac{3+5-2}{3\cdot 5}} + |e_{4,4}|\sqrt{\frac{4+4-2}{4\cdot 4}} + |e_{4,6}|\sqrt{\frac{4+6-2}{4\cdot 6}} + |e_{5,5}|\sqrt{\frac{5+5-2}{5\cdot 5}} + |e_{5,6}|\sqrt{\frac{5+6-2}{5\cdot 6}} + |e_{6,6}|\sqrt{\frac{6+6-2}{6\cdot 6}}$$

$$= 4\sqrt{\frac{3+4-2}{3\cdot 4}} + 8\sqrt{\frac{3+5-2}{3\cdot 5}} + 2(m-3)\sqrt{\frac{4+4-2}{4\cdot 4}} + 4(m-2)\sqrt{\frac{4+6-2}{4\cdot 6}} + 4(n-3)\sqrt{\frac{5+5-2}{5\cdot 5}} + 2(n-2)\sqrt{\frac{5+6-2}{5\cdot 6}}\ (3mn-8m-7n+18)\sqrt{\frac{6+6-2}{6\cdot 6}}$$

$$= \frac{(15\sqrt{6}+40\sqrt{3}-40\sqrt{10})m+(48\sqrt{2}-6\sqrt{30}-35\sqrt{10})n+15\sqrt{10}mn+(20\sqrt{15}+138\sqrt{10}-45\sqrt{6}-80\sqrt{3}-144\sqrt{2})}{30}$$

Case 2 : If $m = 2$ and $n > 2$

In this case cylinder contains $e_{3,3}$, $e_{3,5}$ and $e_{5,5}$ edges. The edges $e_{3,3}$, $e_{3,5}$ and $e_{5,5}$ are colored in red, blue and black respectively as shown in the Figure 8. The number of $e_{3,3}$, $e_{3,5}$ and $e_{5,5}$ edges in each row is mentioned in the following table 3.1.

$\|e_{3,3}\|$	$\|e_{3,5}\|$	$\|e_{5,5}\|$
2	8	$(5n-14)$

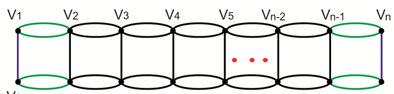

Figure 8

$$ABC(C(2,n)) = \sum_{uv \in E(C(2,n))} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

$$= |e_{3,3}|\sqrt{\frac{3+3-2}{3\cdot 3}} + |e_{3,5}|\sqrt{\frac{3+5-2}{3\cdot 5}} + |e_{5,5}|\sqrt{\frac{5+5-2}{5\cdot 5}}$$

$$= 2\sqrt{\frac{3+3-2}{3\cdot 3}} + 8\sqrt{\frac{3+5-2}{3\cdot 5}} + (5n-14)\sqrt{\frac{5+5-2}{5\cdot 5}}$$

$$= \frac{30\sqrt{2}n + (20 + 24\sqrt{10} - 84\sqrt{2})}{15}$$

Similarly, $ABC(C(m,2)) = \frac{30\sqrt{2}m + (20 + 24\sqrt{10} - 84\sqrt{2})}{15}$

Case 3 : If $m = 2$ and $n = 2$

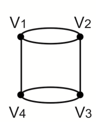

Figure 9

Consider, $ABC(C(2,2)) = |e_{3,3}|\sqrt{\frac{3+3-2}{3\cdot 3}} = 6\sqrt{\frac{3+3-2}{3\cdot 3}} = 4.$ □

Theorem 2.10. *Randic index of cylinder with $(m-1)$ rows and $(n-1)$ columns in each row is given by* $\chi(C(m,n)) =$

$$\begin{cases} \frac{(10\sqrt{6}-25)m + (2\sqrt{30}-11)n + 15mn + (20\sqrt{3} + 16\sqrt{15} - 20\sqrt{6} - 40\sqrt{3} - 27)}{30} & if \quad m > 2\, and n > 2 \\ \frac{15n + (8\sqrt{15} - 32)}{15} & if \quad m = 2\, and n > 2 \\ 2 & if \quad m = 2\, and n = 2 \end{cases}$$

Proof. **Case 1 :** If $m > 2$ and $n > 2$

Randic index of cylinder is $\chi(C(m,n)) = \sum_{uv \in E(\chi(C(m,n)))} \frac{1}{\sqrt{d_u d_v}}$

$$= |e_{3,4}|\frac{1}{\sqrt{3\cdot 4}} + |e_{3,5}|\frac{1}{\sqrt{3\cdot 5}} + |e_{4,4}|\frac{1}{\sqrt{4\cdot 4}} + |e_{4,6}|\frac{1}{\sqrt{4\cdot 6}} + |e_{5,5}|\frac{1}{\sqrt{5\cdot 5}} + |e_{5,6}|\frac{1}{\sqrt{5\cdot 6}} + |e_{6,6}|\frac{1}{\sqrt{6\cdot 6}}$$

$$= 4\frac{1}{\sqrt{3\cdot 4}} + 8\frac{1}{\sqrt{3\cdot 5}} + 2(m-3)\frac{1}{\sqrt{4\cdot 4}} + 4(m-2)\frac{1}{\sqrt{4\cdot 6}} + 4(n-3)\frac{1}{\sqrt{5\cdot 5}} + 2(n-2)\frac{1}{\sqrt{5\cdot 6}}$$

$+(3mn-8m-7n+18)\frac{1}{\sqrt{6\cdot 6}}$
$= \frac{(10\sqrt{6}-25)m+(2\sqrt{30}-11)n+15mn+(20\sqrt{3}+16\sqrt{15}-20\sqrt{6}-40\sqrt{3}-27)}{30}$

Case 2 : If $m=2$ and $n>2$

Consider, $\chi(C(2,n)) = \sum_{uv\in E(\chi(C(2,n)))} \frac{1}{\sqrt{d_u d_v}}$
$= |e_{3,3}|\frac{1}{\sqrt{3\cdot 3}} + |e_{3,5}|\frac{1}{\sqrt{3\cdot 5}} + |e_{5,5}|\frac{1}{\sqrt{5\cdot 5}}$
$= 2\frac{1}{\sqrt{3\cdot 3}} + 8\frac{1}{\sqrt{3\cdot 5}} + (5n-14)\frac{1}{\sqrt{5\cdot 5}}$
$= \frac{15n+(8\sqrt{15}-32)}{15}$

Similarly, $\chi(C(m,2)) = \frac{15m+(8\sqrt{15}-32)}{15}$

Case 3 : If $m=2$ and $n=2$

Consider, $\chi(C(2,2)) = \sum_{uv\in E(\chi(C(2,2)))} \frac{1}{\sqrt{d_u d_v}} = |e_{3,3}|\frac{1}{\sqrt{3\cdot 3}} = \frac{6}{3} = 2$ □

Theorem 2.11. *Sum Connectivity index of Cylinder with $(m-1)$ rows and $(n-1)$ columns in each row is given by $S(C(m,n))$*

$$= \begin{cases} \frac{(\sqrt{2310}+4\sqrt{462}-8\sqrt{385})m+(4\sqrt{462}+4\sqrt{105}-7\sqrt{385})n+3\sqrt{385}mn+(8\sqrt{165}+18\sqrt{385}+\sqrt{2310}-20\sqrt{462}-8\sqrt{105})}{2\sqrt{1155}} & \text{if } m>2 \text{ and } n>2 \\ \frac{5\sqrt{3}n+(2\sqrt{5}+4\sqrt{15}-14\sqrt{3})}{\sqrt{30}} & \text{if } m=2 \text{ and } n>2 \\ \sqrt{6} & \text{if } m=2 \text{ and } n=2 \end{cases}$$

Proof. **Case 1 :** If $m>2$ and $n>2$

Sum Connectivity Index of Cylinder is given by $S(C(m,n)) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u+d_v}}$
$= |e_{3,4}|\frac{1}{\sqrt{3+4}} + |e_{3,5}|\frac{1}{\sqrt{3+5}} + |e_{4,4}|\frac{1}{\sqrt{4+4}} + |e_{4,6}|\frac{1}{\sqrt{4+6}} + |e_{5,5}|\frac{1}{\sqrt{5+5}} + |e_{5,6}|\frac{1}{\sqrt{5+6}} + |e_{6,6}|\frac{1}{\sqrt{6+6}}$
$= 4\frac{1}{\sqrt{3+4}} + 8\frac{1}{\sqrt{3+5}} + 2(m-3)\frac{1}{\sqrt{4+4}} + 4(m-2)\frac{1}{\sqrt{4+6}} + 4(n-3)\frac{1}{\sqrt{5+5}} + 2(n-2)\frac{1}{\sqrt{5+6}}$
$+(3mn-8m-7n+18)\frac{1}{\sqrt{6+6}}$
$= \frac{(\sqrt{2310}+4\sqrt{462}-8\sqrt{385})m+(4\sqrt{462}+4\sqrt{105}-7\sqrt{385})n+3\sqrt{385}mn+(8\sqrt{165}+18\sqrt{385}+\sqrt{2310}-20\sqrt{462}-8\sqrt{105})}{2\sqrt{1155}}$

Case 2 : If $m=2$ and $n>2$

Consider, $S(C(2,n)) = \sum_{uv\in E(C(2,n))} \frac{1}{\sqrt{d_u+d_v}}$
$= |e_{3,3}|\frac{1}{\sqrt{3+3}} + |e_{3,5}|\frac{1}{\sqrt{3+5}} + |e_{5,5}|\frac{1}{\sqrt{5+5}}$
$= 2\frac{1}{\sqrt{3+3}} + 8\frac{1}{\sqrt{3+5}} + (5n-14)\frac{1}{\sqrt{5+5}}$
$= \frac{5\sqrt{3}n+(2\sqrt{5}+4\sqrt{15}-14\sqrt{3})}{\sqrt{30}}$

Similarly, $S(C(m,2)) = \frac{5\sqrt{3}m+(2\sqrt{5}+4\sqrt{15}-14\sqrt{3})}{\sqrt{30}}$

Case 3 : If $m = 2$ and $n = 2$

Consider, $S(C(2,2)) = \sum_{uv \in E(C(2,2))} \frac{1}{\sqrt{d_u + d_v}} = |e_{3,3}| \frac{1}{\sqrt{3+3}} = 6\frac{1}{\sqrt{6}} = \sqrt{6}.$ □

Theorem 2.12. *Geometric-arithmetic index of cylinder with $(m-1)$ rows and $(n-1)$ columns in each row is given by* $GA(C(m,n)) =$

$$\begin{cases} \frac{(616\sqrt{6}-2310)m+(140\sqrt{30}-1155)n+1155mn+(880\sqrt{3}+770\sqrt{15}-1232\sqrt{6}-280\sqrt{30})}{385} & if \quad m > 2 \, and \, n > 2 \\ 5n + (2\sqrt{15} - 12) & if \quad m = 2 \, and \, n > 2 \\ 6 & if \quad m = 2 \, and \, n = 2 \end{cases}$$

Proof. **Case 1 :** If $m > 2$ and $n > 2$

Geometric-Arithmetic Index of cylinder is given by $GA(C(m,n)) = \sum_{e=uv \in E(C(m,n))} \frac{2\sqrt{d_u d_v}}{d_u d_v}.$

$$= |e_{3,4}|\frac{2\sqrt{3\cdot 4}}{3+4} + |e_{3,5}|\frac{2\sqrt{3\cdot 5}}{3+5} + |e_{4,4}|\frac{2\sqrt{4\cdot 4}}{4+4} + |e_{4,6}|\frac{2\sqrt{4\cdot 6}}{4+6} + |e_{5,5}|\frac{2\sqrt{5\cdot 5}}{5+5} + |e_{5,6}|\frac{2\sqrt{5\cdot 6}}{5+6} + |e_{6,6}|\frac{2\sqrt{6\cdot 6}}{6+6}$$
$$= 4\frac{2\sqrt{3\cdot 4}}{3+4} + 8\frac{2\sqrt{3\cdot 5}}{3+5} + 2(m-3)\frac{2\sqrt{4\cdot 4}}{4+4} + 4(m-2)\frac{2\sqrt{4\cdot 6}}{4+6} + 4(n-3)\frac{2\sqrt{5\cdot 5}}{5+5} + 2(n-2)\frac{2\sqrt{5\cdot 6}}{5+6}$$
$$+(3mn - 8m - 7n + 18)\frac{2\sqrt{6\cdot 6}}{6+6}$$
$$= \frac{(616\sqrt{6}-2310)m+(140\sqrt{30}-1155)n+1155mn+(880\sqrt{3}+770\sqrt{15}-1232\sqrt{6}-280\sqrt{30})}{385}$$

Case 2 : If $m = 2$ and $n > 2$

Consider, $GA(C(2,n)) = \sum_{uv \in E(C(2,n))} \frac{2\sqrt{d_u d_v}}{d_u + d_v}.$

$$= |e_{3,3}|\frac{2\sqrt{3\cdot 3}}{3+3} + |e_{3,5}|\frac{2\sqrt{3\cdot 5}}{3+5} + |e_{5,5}|\frac{2\sqrt{5\cdot 5}}{5+5}$$
$$= 2\frac{2\sqrt{3\cdot 3}}{3+3} + 8\frac{2\sqrt{3\cdot 5}}{3+5} + (5n-14)\frac{2\sqrt{5\cdot 5}}{5+5}$$
$$= 5n + (2\sqrt{15} - 12)$$

Similarly, $GA(C(m,2)) = 5m + (2\sqrt{15} - 12)$

Case 3 : If $m = 2$ and $n = 2$

Consider, $GA(C(2,2)) = \sum_{uv \in E(C(2,n))} \frac{2\sqrt{d_u d_v}}{d_u + d_v} = |e_{3,3}|\frac{2\sqrt{3\cdot 3}}{3+3} = 6$ □

Theorem 2.13. *Atom bond connectivity index of torus with $(m-1)$ rows and $(n-1)$ columns in each row is given by* $ABC(T(m,n)) = mn\sqrt{\frac{3}{2}}$

Proof. The topological structure of a torus network is denoted $T(m,n)$ and is defined as the cartesian product $C_m \times C_n$ where C_m and C_n are undirected cycles. The numbering adopted for torus is same as that of grid.

Consider a Torus with $(m-1)$ rows and $(n-1)$ columns in each row. Let $e_{i,j}$ denotes the number of edges connecting the vertices of degrees d_i and d_j. Two-dimensional structure of Torus is as shown in the Figure 10 contains only $e_{4,4}$ edges. The number of $e_{4,4}$ edges in each row is mentioned in the following table - 4.

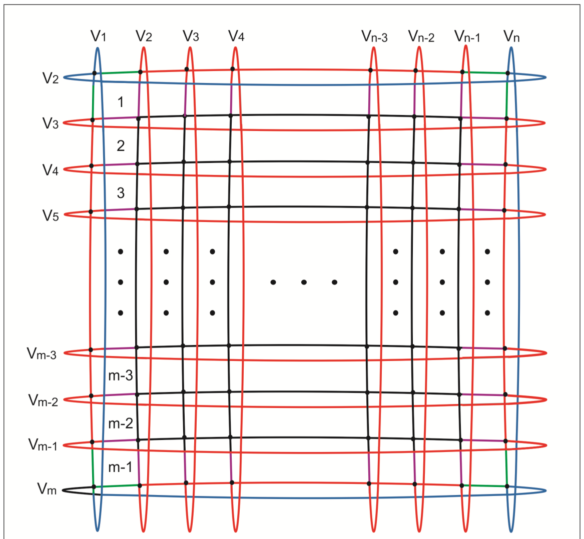

Figure 10

Row	$\|e_{4,4}\|$
1	3n
2	2n
3	2n
4	2n
. . .	. . .
$m-2$	2n
$m-1$	3n
Total	2mn

$\therefore |e_{4,4}| = 2mn$ edges.

Consider, $ABC(T(m,n)) = \sum_{uv \in E(T(m,n))} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$

$ABC(T(m,n)) = |e_{4,4}|\sqrt{\frac{4+4-2}{4 \cdot 4}} = 2mn\sqrt{\frac{4+4-2}{4 \cdot 4}} = \sqrt{\frac{3}{2}}mn.$

□

Theorem 2.14. *Randic index of torus with $(m-1)$ rows and $(n-1)$ columns in each row is given by* $\chi(T(m,n)) = \frac{mn}{2}$

Proof. Randic index of torus is $\chi(T(m,n)) = \sum_{uv \in E(\chi(T(m,n)))} \frac{1}{\sqrt{d_u d_v}}$

$$\chi(T(m,n)) = |e_{4,4}| \frac{1}{\sqrt{4 \cdot 4}} = 2mn \frac{1}{\sqrt{4 \cdot 4}} = \frac{2mn}{4} = \frac{mn}{2} \qquad \square$$

Theorem 2.15. *Sum connectivity index of torus with* $(m-1)$ *rows and* $(n-1)$ *columns in each row is given by* $S(T(m,n)) = \frac{mn}{\sqrt{2}}$

Proof. Sum connectivity index of torus is given by $S(T(m,n)) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u + d_v}}$

$$S(T(m,n)) = |e_{4,4}| \frac{1}{\sqrt{4+4}} = 2mn \frac{1}{\sqrt{4+4}} = \frac{2mn}{\sqrt{8}} = \frac{mn}{\sqrt{2}} \qquad \square$$

Theorem 2.16. *Geometric-arithmetic(GA) index of Torus with* $(m-1)$ *rows and* $(n-1)$ *columns in each row is given by* $\chi(T(m,n)) = 2mn$

Proof. Geometric-arithmetic(GA) index of Torus is given by $GA(T(m,n)) = \sum_{e=uv \in E(T(m,n))} \frac{2\sqrt{d_u d_v}}{d_u + d_v}$

$$= |e_{4,4}| \frac{2\sqrt{4 \cdot 4}}{(4+4)} = 2mn \frac{2\sqrt{4 \cdot 4}}{4+4} = 2mn \qquad \square$$

3 Conclusion

The problem of finding the general formula for ABC index, Randic connectivity index, Sum connectivity index and GA index of grid, extended grid, torus and cylinder is solved here analytically without using computers.

Conflict of interests. The authors declares that there is no conflicts of interests regarding the publication of this paper.

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