New method to determine effective atomic number of samples via external bremsstrahlung

M. V. Manjunatha, B. M. Sankarshan and T. K. Umesh*

In the present investigation, the variation in the external bremsstrahlung (EB) intensity produced due to the beta particles emitted by the $^{90}$Sr-$^{90}$Y source in the elements Al, Cu, Ag, Sn, and Pb as well as some lead compounds was studied as a function of their masses per unit area. By a suitable regression analysis, two new, handy, and simple expressions for the effective atomic number of the radiator were derived in terms of the measured EB intensity. The novelty of the present method is that these expressions require either two different samples of the same mass per unit area or a single sample of two different masses per unit area for determining the effective atomic number. To check the efficacy of the method, from the measured EB intensity of the group of lead compounds, their effective atomic number was determined by making use of these expressions. The results were found to be in good agreement with the modified atomic number $Z_{\text{mod}}$ calculated by using the theoretical expression of Markowicz and Van Grieken. This establishes the fact that this new method employs simple and very handy expressions which will be convenient particularly in the case of samples which are not abundantly available in nature.

Introduction

It is often found convenient to represent the interaction properties of a composite material by an effective atomic number $Z_{\text{eff}}$. This number depends on the incident energy as well as the atomic number of the constituent elements.$[11]$ It indicates on an average the number of electrons of the material that actively participate in the photon atom interaction.$[2]$ Several experimental techniques such as chemical analysis, atomic absorption spectroscopy, particle induced X-ray emission (PIXE), X-ray fluorescence (XRF), proton induced gamma ray emission (PIGE), fast neutron activation analysis (FNAA), acoustic method, electrical impedance method and Rayleigh to Compton scattering ratio method are in vogue to determine $Z_{\text{eff}}$ of composite material like a mixture or a compound.$[3-5]$ $Z_{\text{eff}}$ can also be determined by measuring the external bremsstrahlung (EB) intensity.

External bremsstrahlung is a process which is usually accompanied by the emission of continuous electromagnetic radiation. It involves the Coulomb interaction of the atomic nuclei with the incident beta particles from a radioactive source. Whenever a beta particle has only one encounter during its passage through the material, the resulting EB process is called ‘thin target bremsstrahlung’. However in situations where the target has a thickness which is a significant fraction of the mean range of the beta particles in its medium, the resulting EB process is called ‘thick target bremsstrahlung’.

External bremsstrahlung is known to be a process of radiative origin. Hence, it shows a characteristic dependence on the atomic number $Z$ of the target (radiator). Several investigators$[6-9]$ have confirmed based on their study that the thick target EB intensity per atom from different radiators varies to a good approximation as $Z^2$ over the entire continuous energy spectrum of the beta source employed. This aspect could be used to advantage in determining the effective atomic number of any composite material such as a compound or an alloy, if either its composition or molecular weight is known. In this context, it is worth noting that EB measurements can be carried out even with fairly thick radiators and a weak beta source. This is practically more advantageous than a photon attenuation experiment which needs a much stronger source.$[10]$ This has prompted several investigators$[11-13]$ to measure the EB intensity in elements and other samples and use it to determine the effective atomic number $Z_{\text{eff}}$ of these samples. The $Z_{\text{eff}}$ values have been usually compared with the modified mean atomic number $Z_{\text{mod}}$ given by the Markowicz and Van Grieken equation.$[14]$ In the determination of the effective atomic number, the EB intensity produced due to the interaction of the incident beta particles with a monoatomic layer of the target of interest is used. This necessitates a measurement of the EB intensity produced due to samples of different thicknesses (masses per unit area) of the same target. Although most of the elements are abundantly available in their foil form, a vast majority of the composite targets such as compounds, alloys, rare earth samples, and biological samples are scarce in nature. As a consequence, it may be difficult to obtain sufficient number of such samples of varying thickness. This is expected to impose a serious practical limitation. In view of this, it was felt worthwhile by us to evolve a new method which does not require more than two thicknesses of the same sample to determine the effective atomic number. In this method, expressions have been obtained

* Correspondence to: T. K. Umesh, Department of studies in Physics, University of Mysore, Manasagangotri, Mysore, Karnataka 570006, India.
E-mail: tku@physics.uni-mysore.ac.in

Department of studies in Physics, University of Mysore, Manasagangotri, Mysore, Karnataka 570006, India
which require either two different samples of the same thickness or one sample of two different thicknesses.

In the present work, the efficacy of this new method is illustrated by determining the effective atomic number of some lead compounds based on the measured EB intensity using a weak $^{90}\text{Sr},^{90}\text{Y}$ beta source. The $Z_{\text{eff}}$ values so obtained are compared with the $Z_{\text{mod}}$ values calculated by using the Markowicz and Van Grieken equation as well as those obtained from the conventional intercept method and discussed.

**Theory**

According to Mudhole and Umakantha, the total intensity $I$ of thick target EB produced by beta particles in a target of atomic (molecular) weight $A$ and mass per unit area $\rho t$ (after including the attenuation of the EB X-rays in the target material) is given by the empirical relation:

$$I / N = KZ^n e^{-\xi \rho t}$$  \hspace{1cm} (1)

where $Z$ is the atomic number of the target material, $K$ is a proportionality constant which depends on the energy and intensity of the $\beta$ emitter, the bremsstrahlung cross section and geometry of the experimental setup and is independent of the thickness of the target. $\xi$ is the mass attenuation coefficient and $\rho t$ is the mass per unit area of the target such that $\rho t \leq 0.4R$ where $R$ is the range of beta particles in the target and $N = \frac{4\pi t}{A}$ is the number of atoms (molecules) per unit area in the target. Eqn (1) implies that when the bremsstrahlung X-rays progress through the target, they encounter several monoatomic layers of the target material and will suffer an exponential attenuation in it. The extent of this attenuation in a given target is represented by the mass attenuation coefficient $\xi$. Clearly, this quantity is independent of the mass per unit area of the target but depends on its atomic number. In terms of natural logarithms, Eqn (1) can be written as

$$\ln \left( \frac{I}{N} \right) = \ln(KZ^n) - \left( \frac{\xi}{\rho} \right) pt$$  \hspace{1cm} (2)

Equation (2) represents a straight line with $\xi/\rho$ as the slope and $\ln(KZ^n)$ as the intercept, respectively. The intercept $\ln(KZ^n)$ represents the EB intensity per monoatomic layer of the target. By experimentally determining $\xi/\rho$ for different masses per unit area $\rho t$ of the same sample, it is possible to obtain a straight line of the form Eqn (2) for each sample of interest from which the best fit values of the slope and the intercept $\ln(KZ^n)$ can be obtained by a proper linear regression analysis. By a definition of the natural logarithm, we have

$$\ln(KZ^n) = \ln K + n \ln Z$$  \hspace{1cm} (3)

Equation (3) is an equation for a straight line with $n$ as the slope and $\ln K$ as the intercept, respectively. By a suitable regression analysis, the best fit values of $n$ and $\ln K$ may be determined which are constants for the given geometrical set up and targets used. If the values of $n$ and $\ln K$ are determined for elements of known $Z$, the $Z_{\text{eff}}$ of the unknown samples may be determined from the measured EB intensity by using Eqns (2) and (3) (using the intercept $\ln(KZ^n)$). This is nothing but the conventional intercept method. Obviously, this requires several thicknesses of the same sample for determining its $Z_{\text{eff}}$. The value of $\frac{\xi}{\rho} n$, and $\ln K$ may be used to determine the effective atomic number of the sample by a novel approach which is as follows.

Although, the energy spectrum of the beta source and EB photons is continuous, practically in EB measurements it is considered to have an average energy of $(0.4 \pm 0.05) E_0$, where $E_0$ is the end point energy. In the present work, a $^{90}\text{Sr},^{90}\text{Y}$ source of end point energy $E_0 = 2.270 \text{MeV}$ was used. This corresponded to an average of $(0.908 \pm 0.114) \text{MeV}$. At this energy on an average, the principle mode of photon attenuation is the incoherent scattering process which is linearly dependent on $Z$. Therefore, the values of the mass attenuation coefficient $\xi$ show a linear dependence on the atomic number of the target, in the energy region of interest. It is clear that $\ln(KZ^n)$ and $\frac{\xi}{\rho}$ are the intercept and slope values of a linear plot of $\ln(\frac{I}{N})$ versus $\rho t$ for a given $Z$. By definition of logarithm, $\ln(KZ^n)$ varies linearly with $\ln Z$. Hence, without loss of generality and for ease of further calculation, the values of $\frac{\xi}{\rho}$ were also expected to show a linear variation with $\ln Z$. This is a straight line of the form

$$\sum_{\rho} = C + D \ln Z$$  \hspace{1cm} (4)

where $C$ and $D$ are the intercept and slope, respectively. By using Eqns (4) and (3) in Eqn (2), we have

$$\ln \left( \frac{I}{N} \right) = \ln K + n \ln Z - \left[ C + D \ln Z \right] pt$$  \hspace{1cm} (5)

Suppose we have two targets of different composition and same mass per unit area of which one is an element of atomic number $Z$ and the other is a target whose $Z_{\text{eff}}$ is to be determined, from Eqn (5), we can write,

$$\ln \left( \frac{I_1}{N_1} \right) = \ln K + n \ln Z - \left[ C + D \ln Z \right] pt$$  \hspace{1cm} (6)

$$\ln \left( \frac{I_2}{N_2} \right) = \ln K + n \ln Z_{\text{eff}} - \left[ C + D \ln Z_{\text{eff}} \right] pt$$  \hspace{1cm} (7)

Using Eqn (6) and Eqn (7), we obtain

$$Z_{\text{eff}} = \exp \left\{ \ln Z - \left[ \frac{\ln(\frac{I_1}{N_1}) - \ln(\frac{I_2}{N_2})}{n - D pt} \right] \right\}$$  \hspace{1cm} (8)

By using Eqn (8), $Z_{\text{eff}}$ may be determined if $Z$ is known provided the two targets have the same mass per unit area $\rho t$. We shall call this method as the 'two sample method'. Interestingly, the requirement of two samples in this method could be eliminated by the following procedure: suppose we have a sample of unknown $Z_{\text{eff}}$ available with two different masses per unit area $(\rho t)_1$ and $(\rho t)_2$, we can rewrite Eqn (5) for this case as

$$\ln \left( \frac{I}{N} \right)_1 = \ln K + n \ln Z_{\text{eff}} - \left[ C + D \ln Z_{\text{eff}} \right] (\rho t)_1$$  \hspace{1cm} (9)

$$\ln \left( \frac{I}{N} \right)_2 = \ln K + n \ln Z_{\text{eff}} - \left[ C + D \ln Z_{\text{eff}} \right] (\rho t)_2$$  \hspace{1cm} (10)
By using Eqns (9) and (10), we can write

\[
Z_{\text{eff}} = \exp \left\{ \frac{1}{D} \left[ \frac{\ln \left( \frac{I}{N_1} \right)}{\ln \left( \frac{I}{N_2} \right)} - C \right] \right\}
\]

(11)

By using Eqn (11), we can determine the effective atomic number \(Z_{\text{eff}}\) of the unknown target by using the measured bremsstrahlung intensity due to two different thicknesses. Let us call this method as one sample method. The \(Z_{\text{eff}}\) values obtained from the experimenaly measured EB intensity of elements and compounds are usually compared with those obtained from the expressions of the modified effective atomic number of a compound or mixture given by the theoretical expression of Markowicz and Van Grieken.

\[
Z_{\text{mod}} = \sum_{i=1}^{l} \frac{w_i'Z_i}{A_i}
\]

(12)

where \(i\) is the number of elements in the compound or mixture, and \(W_i', A_i,\) and \(Z_i\) are the weight fraction, the atomic weight, and atomic number of the \(i\)th element in the target, respectively. The modified effective atomic number \(Z_{\text{mod}}\) is defined in such a way that the yield of a mixture or chemical compound agrees with that of a pure element with \(Z = Z_{\text{mod}}\). This expression of \(Z_{\text{mod}}\) is known to be applicable for bremsstrahlung processes in compounds, mixtures, and alloys.

**Experimental details**

The experimental arrangement used in the present investigation is as shown in Fig. 1. In the experiment, a \(^{90}\text{Sr}^{90}\text{Y}\) pure beta source (\(2\pi\)Ci, half life 28 years) kept in a Perspex holder served as the source of beta particles. All sides of the beta source were covered by Perspex so as to stop beta particles in all directions except in the forward direction and there by minimize extraneous EB production from the surrounding material. A 120-mm thick Perspex sheet is kept between the source and the detector so as to stop beta particles in all directions except in the forward direction and thereby minimize extraneous EB production from the surrounding material as well as the background. The EB produced in the Perspex is expected to be negligible compared to that produced in higher Z targets used in the experiment. The difference of the corresponding intensities in \(P_1\) and \(P_2\) positions gives the EB intensity \(I\) produced in the target by the incident beta particles. Experimentally, values of \(I\) were determined for several masses per unit area of a given sample.

Because a beta stopper (Perspex) of sufficient thickness was introduced between the positions \(P_1\) and \(P_2\), no beta particle could reach the target kept in position \(P_2\). Hence, EB was not produced in the target in this position. So, the intensity measured by keeping the target in position \(P_2\) was the sum of the intensities due to IB from the source attenuated in the same thickness (mass per unit area) of the target as well as the background.

The EB produced in the Perspex is expected to be negligible compared to that produced in higher Z targets used in the experiment. The difference of the corresponding intensities in \(P_1\) and \(P_2\) positions gives the EB intensity \(I\), produced in the target by the incident beta particles. Experimentally, values of \(I\) were determined for several masses per unit area of a given sample.

It is clear from the previous (Theory) section that techniques described in the present work need the best fit values of \(lnK, n, C,\) and \(D\) for determining \(Z_{\text{eff}}\). These quantities for the present geometrical set up were determined from the measured EB intensities for the elements of Z in the range 13–82 by employing the difference method described earlier. For this purpose, the experiment was conducted for the elements Al, Cu, Ag, Sn, and Pb with 99.9% pure foils. The beta source was placed in a Perspex stand at a distance of 12.5 cm above the face of the detector. The target was placed suitably between the source and detector alternatively in the \(P_1\) and \(P_2\) positions. The measurements were carried out for different thicknesses (masses per unit area) of the elemental foils. The geometry was carefully adjusted to see that crystal was fully exposed to the EB emitted from the target.

Target compounds used in the present study were a group of lead compounds. These compounds in fine powder form were
filled in Perspex planchets of 1 cm diameter. A number of such planchets were prepared. The masses per unit area \( \rho t \) of these compounds were so chosen that \( \rho t \leq 0.4R \) where \( R \) is the range of the beta particles in that material.

Errors

The errors involved in the experiment were mostly due to counting statistics. In the experiment, about \( 10^3 \)–\( 10^5 \) counts were recorded. Thus, the error due to counting statistics was kept below 1 \%

Each compound in fine powder form was confined in cylindrical Perspex containers. The mass of the compound was determined in an electrical balance correct to the third decimal place. The inner diameter of the container was determined with the aid of a travelling microscope. From these, the mass per unit area was calculated for each sample. The uncertainty in the mass per unit area of the samples was found to be less than 0.1 \% in all cases.

The error due to sample impurity could be large only when a large percentage of high \( Z \) impurities is present in the sample. Because the samples themselves contained a high \( Z \) element, the error due to sample impurities was found to be negligible in the present investigation.

It may be argued that whenever thicker targets are used, the target provides for an effective mean free path for the photons to scatter and rescatter inside its medium resulting in a net photon dose build up inside it. Clearly, this is a function of the target thickness, \( Z \) as well as the incident intensity. In the present experiment, a weak beta source was employed together with optimal target thickness. Also, the final calculations were made on the intensity values for zero target thickness. Hence, the multiple scattering effects were considered to be negligible.

The error due to non-uniformity of the sample was found to be negligible by exposing different parts of the same sample to the incident beam and repeating the measurements every time. Thus overall error in the present measurement was expected to be less than 2 \%.

It is worth noting that spurious counts arising from Photo multiplier tube noise may interfere with the intensity measurements. All intensity measurements were made by fixing the base level (LLD level) in the MCA corresponding to about 20 keV. In this setting, the recorded counts did not include the low energy noise contribution.

The background radiation could be a combination of the room background as well as the EB produced in the perspex \( \beta \)-stopper. The room background was kept to a minimum by removing all radioactive sources except the \(^{90}\text{Sr}^{90}\text{Y}\) source from the laboratory. The EB produced in the perspex was low in intensity and energy. The small background intensity was eliminated in the measurement because of the difference method adopted in the present work.

Results and discussion

A typical resultant spectrum of lead which is obtained by subtracting the spectrum obtained in the \( P_2 \) position from that obtained in the \( P_1 \) position is as shown in Fig. 2. This is the EB spectrum of lead. Similarly, the EB spectra were obtained for all the other elements and Pb compounds of interest. By using these spectra, the values of \( \ln(\beta) \) were calculated for all the samples. The \( \ln(\beta) \) values of the elements were plotted as a function of their respective masses per unit area. The resulting plots as shown in Fig. 3 were found to be linear according to Eqn (3).

By a suitable regression analysis, the best fit values of the slopes \( \frac{1}{\rho} \) and intercepts \( \ln(KZ^n) \) were determined. Further, the \( \ln(KZ^n) \) values of the elemental targets were plotted versus \( \ln Z \). The resulting plot was a straight line as shown in Fig. 4 with \( \ln K \) as the intercept and \( n \) as the slope as per Eqn (4). The best fit values of \( \ln K \) and \( n \) for the source, geometry, and target combination were determined by a linear least square fitting. It was found that \( n \) was close to 2. The values of \( \ln K \) and \( \frac{1}{\rho} \) were obtained by fixing the value of \( n \) as 2. The value of \( \ln K \) was obtained as \(-36.125 \pm 0.112 \).

Further, the fit values of \( \frac{1}{\rho} \) so obtained were plotted as a function of \( \ln Z \) along with the error bars. This plot was a straight line as shown in Fig. 5 with \( C \) and \( D \) as intercept and slope, respectively. By a suitable linear regression analysis, the best fit values of the coefficients \( C \) and \( D \) were determined. These values were found to be \(-0.356 \pm 0.038 \) and \( 0.792 \pm 0.011 \), respectively. These were subjected to a \( \chi^2 \)-test. For this purpose, the values of \( \ln K, C, \) and \( D \) were varied in small steps within their respective standard deviations, and the fit values of \( Z_{\text{eff}} \) of the elements...
Within the stated uncertainties. This increased confidence in the method employed.

Further, from \( \ln(\chi) \) values of the different lead compounds, their \( Z_{\text{eff}} \) values were evaluated from the best fit values of \( \ln K, n, C, \) and \( D \) by the intercept method, one sample method, and the two sample method. These values are shown in Table 1 along with the estimated uncertainties and compared with the theoretical values obtained from Eqn (12) for \( Z_{\text{mod}} \).

The uncertainty on \( Z_{\text{eff}} \) was mainly a result of experimental errors as well as errors involved during fitting procedure. The uncertainty budget was estimated for the three methods separately as detailed below:

**Intercept method**

The uncertainty on \( Z_{\text{eff}} \) determined by using this method is decided by how accurately the intercept \( \ln(KZ^n) \) could be determined from a plot of \( \ln(\chi) \) versus \( pt \) for a given target. Obviously, this is governed primarily by the experimental error on \( l \) and the fitting error on \( \ln K \).

In the present measurement, as discussed earlier, the experimental error on \( l \) was expected to be less than 2%. For our purpose, we shall assume that it was 2%. Plots of \( \ln(\chi) \) versus \( pt \) were prepared for all samples of interest (elements and compounds) along with these error bars. A typical plot for Pb is as shown in Fig. 3.

To estimate the percentage uncertainty \( e_{\text{exp}} \) on \( Z_{\text{eff}} \) due to the experimental error bars on \( \ln(\chi) \), two hypothetical lines were drawn by connecting the points:

(a) \( \ln(\frac{1.02I}{N}) \) at \( (\rho I)_{\text{min}} \) and \( \ln(\frac{0.98I}{N}) \) at \( (\rho I)_{\text{max}} \) and

(b) \( \ln(\frac{0.98I}{N}) \) at \( (\rho I)_{\text{min}} \) and \( \ln(\frac{1.02I}{N}) \) at \( (\rho I)_{\text{max}} \)

where \( (\rho I)_{\text{min}} \) and \( (\rho I)_{\text{max}} \) were the minimum and maximum thicknesses of the particular target, respectively, used for the experiment and 1.02I and 0.98I correspond to \( (l \pm 2\% \text{ of } l) \), respectively.

This was performed for all five elements of interest to the present work (13 < \( Z < 82 \)). The slopes \( (m_j) \), and \( (m_j) \) as well as the intercepts \( (c_j) \) and \( (c_j) \) for the \( i^{th} \) element determined in this manner were their limiting (extremum) values. Clearly, \( (c_j) \) and \( (c_j) \) were the \( \ln(KZ^n) \) values for a given \( Z \). These values were separately used in Eqn (3) to determine the \( Z_{\text{eff}} \) values of the elements Al, Cu, Ag, Sn, and Pb which were then compared with their known values (13, 29, 47, 50, and 82). This indicated that the maximum percentage uncertainty \( e_{\text{exp}} \) on \( Z_{\text{eff}} \) was 1.5. Because the \( Z_{\text{eff}} \) of a given sample is known to be always less than or equal to the highest \( Z \) among its constituent elements (in this case Pb), the maximum percentage uncertainty \( e_{\text{exp}} \) due to experimental error in the intercept method was expected to be 1.5 for all samples.

The percentage uncertainty \( e_{\text{fit}} \) on \( Z_{\text{eff}} \) due to fitting error was a result of the standard deviation on the best fit value of \( \ln K \). The values of \( \ln K \) were varied in small steps within the standard deviation range and every time the \( Z_{\text{eff}} \) was calculated for the chosen value of \( \ln K \), using Eqn (3) for the elements of interest. In this way, the highest percentage uncertainty \( e_{\text{fit}} \) on \( Z_{\text{eff}} \) was found to be 5.62. The r.m.s. value of the maximum percentage uncertainty on \( Z_{\text{eff}} \) based on the intercept method was now calculated according to...
This was found to be 5.82.

**Two sample method**

The two sample method unlike the intercept method is a method involving comparison of intensities measured for two different samples, at the same thickness. Hence, in order to estimate the percentage uncertainties on $Z_{\text{eff}}$ due to experimental and fitting errors, we use the following equation:

$$e_{\text{rms}} = \pm \left( e_{\text{expt}}^2 + e_{\text{fit}}^2 \right)^{1/2} \quad (14)$$

This was found to be 5.82.

**Table 2.** Percentage uncertainties on $Z_{\text{eff}}$ due to experimental and fitting errors

<table>
<thead>
<tr>
<th>Method</th>
<th>$e_{\text{expt}}$</th>
<th>$e_{\text{fit}}$</th>
<th>$e_{\text{rms}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept method</td>
<td>1.5</td>
<td>5.62</td>
<td>5.82</td>
</tr>
<tr>
<td>One Sample method</td>
<td>—</td>
<td>5.42</td>
<td>5.42</td>
</tr>
<tr>
<td>Two Sample method</td>
<td>—</td>
<td>1.23</td>
<td>1.23</td>
</tr>
</tbody>
</table>

**Table 1.** Intercept $\ln(KZ^n)$ and the effective atomic number $Z_{\text{mod}}$ of samples obtained by different methods

<table>
<thead>
<tr>
<th>Samples</th>
<th>Atomic weight/molecular weight</th>
<th>$\ln(KZ^n)$</th>
<th>$Z_{\text{mod}}$</th>
<th>Effective atomic number($Z_{\text{eff}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Intercept method</td>
</tr>
<tr>
<td>Al</td>
<td>26.982</td>
<td>$-31.105$</td>
<td>13.0</td>
<td>$13.0 \pm 0.76$</td>
</tr>
<tr>
<td>Cu</td>
<td>63.546</td>
<td>$-29.500$</td>
<td>29.0</td>
<td>$29.0 \pm 1.69$</td>
</tr>
<tr>
<td>Ag</td>
<td>107.87</td>
<td>$-28.535$</td>
<td>47.0</td>
<td>$47.0 \pm 2.74$</td>
</tr>
<tr>
<td>Sn</td>
<td>118.71</td>
<td>$-28.411$</td>
<td>50.0</td>
<td>$50.0 \pm 2.91$</td>
</tr>
<tr>
<td>Pb</td>
<td>207.20</td>
<td>$-27.422$</td>
<td>82.0</td>
<td>$82.0 \pm 4.77$</td>
</tr>
<tr>
<td>Lead acetate</td>
<td>379.33</td>
<td>$-28.746$</td>
<td>42.1</td>
<td>$42.3 \pm 2.46$</td>
</tr>
<tr>
<td>Lead bromate</td>
<td>463.00</td>
<td>$-28.498$</td>
<td>47.8</td>
<td>$47.9 \pm 2.79$</td>
</tr>
<tr>
<td>Lead nitrate</td>
<td>331.08</td>
<td>$-28.408$</td>
<td>50.0</td>
<td>$50.1 \pm 2.91$</td>
</tr>
<tr>
<td>Lead formate</td>
<td>297.20</td>
<td>$-28.215$</td>
<td>55.1</td>
<td>$55.1 \pm 3.21$</td>
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<tr>
<td>Lead phosphate</td>
<td>811.54</td>
<td>$-27.971$</td>
<td>62.2</td>
<td>$62.3 \pm 3.63$</td>
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<td>Lead sulphate</td>
<td>303.281</td>
<td>$-27.851$</td>
<td>66.5</td>
<td>$66.2 \pm 3.85$</td>
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<td>Lead dioxide</td>
<td>239.19</td>
<td>$-27.743$</td>
<td>69.9</td>
<td>$69.8 \pm 4.06$</td>
</tr>
</tbody>
</table>

**Figure 6.** $Z_{\text{eff}}$ versus $Z_{\text{mod}}$ for elements and compounds with error bars for intercept method.

**Figure 7.** $Z_{\text{eff}}$ versus $Z_{\text{mod}}$ for elements and compounds with error bars for One sample method.

**Figure 8.** $Z_{\text{eff}}$ versus $Z_{\text{mod}}$ for elements and compounds with error bars for Two sample method.
percentage uncertainty on $Z_{\text{eff}}$ due to experimental error bars (of 2%), two corresponding extremum values of $I_{	ext{Al}}(1.021/N)$ and $I_{\text{Pb}}(0.981/N)$ were used to calculate $\ln \frac{I_{\text{Al}}(1.021/N)}{I_{\text{Pb}}(0.981/N)}$ as well as $\ln \frac{I_{\text{Al}}(0.981/N)}{I_{\text{Pb}}(0.981/N)}$ at a given $\rho t$ for two elements Al and Pb. The $Z_{\text{eff}}$ values were determined accordingly, using Eqn (8) with the best fit value of $D$. The values obtained were compared with the known values of 13 and 82 for Al and Pb. This yielded a percentage uncertainty $e_{\text{expt}}$ which was very small ($=10^{-14}$). Because all samples of interest to the present work were expected to have a $Z_{\text{eff}}$ in the range 13 to 82, $e_{\text{expt}}$ was expected to be negligible in the two sample method.

The percentage uncertainty $e_{\text{err}}$ on $Z_{\text{eff}}$ in the two sample method was solely due to the standard deviation on $D$. The value of $D$ was varied in small steps within its standard deviation for the values of $\ln \left( \frac{1}{N} \right)$ for a chosen thickness $\rho t$ of the elements Al and Pb. The $Z_{\text{eff}}$ values so obtained were compared with their standard values of 13 and 82. By this, it was found that the maximum percentage uncertainty $e_{\text{err}}$ on $Z_{\text{eff}}$ in the two sample method was 1.23. Thus, it was concluded that the r.m.s. percentage uncertainty $e_{\text{rms}}$ on $Z_{\text{eff}}$ did not exceed 1.23 in the two sample method.

One sample method

The one sample method also involves a comparison of two intensities measured for two different thicknesses of the same sample. Hence, the $e_{\text{expt}}$ was estimated by a method similar to the one described for the two sample method excepting that here two different chosen thicknesses for the same elemental sample were considered. This was performed for all elements of interest to the present work. It was found that $e_{\text{expt}}$ was negligible.

Further, the values of $C$ and $D$ were varied within their standard error bars and the values of $Z_{\text{eff}}$ of all elements of interest were calculated from $\ln \left( \frac{1}{N} \right)$ values at a chosen value of $\rho t$ by using Eqn (11). The maximum percentage uncertainty $e_{\text{err}}$ on $Z_{\text{eff}}$ was determined by comparing it with the corresponding known values of $Z$ to be 5.46.

Thus, the maximum percentage uncertainty on the $Z_{\text{eff}}$ values estimated to be 5.82, 1.23, and 5.46 for the intercept method, two sample method, and one sample method, respectively, by using Eqn (14). The values of $e_{\text{expt}}$, $e_{\text{err}}$, and $e_{\text{rms}}$ for the three methods used in the present work calculated in this manner are provided in Table 2.

The $Z_{\text{eff}}$ values of the elements and lead compounds obtained from all the three methods along with their uncertainties are shown in Table 1. Further, these values of $Z_{\text{eff}}$ were plotted together with their uncertainties versus their corresponding $Z_{\text{mod}}$ values. The plots were as shown in Figs. 6–8, respectively.

It can be noticed from the figures that there is a good agreement among the two sets of values within the range of uncertainty in all the three methods.

Conclusions

Thus, in this study, two novel methods have been evolved along with the conventional intercept method which can be conveniently employed to determine the $Z_{\text{eff}}$ of any sample of known composition or molecular weight. Out of the three methods, the two sample method yielded $Z_{\text{eff}}$ values with the lowest percentage uncertainty. This implies that the two sample method is the most suitable method to determine the effective atomic number of any sample in the range $Z_{\text{eff}} = 13$ to 82. This method employs the EB intensity produced in a sample with a weak beta source and a suitable geometrical set up, in two samples of identical masses per unit area. To the knowledge of the authors, the methods described here to determine $Z_{\text{eff}}$ are reported for the first time in literature.

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